# Computer science 

## Python and Latex 2020-2021

Lama TARSISSI

## Outline

## Session I

(1) Introduction
(2) Installation and configuration
(0) Simple examples

## 1.Introduction

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- The first version was published in 1991.
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- Python Software Foundation is the association that organizes the developement of this language and manages the community of developers and users.

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All theses caracteristics make of PYTHON a very useful language.
That's why, nowadays it is used in high schools and higher education levels.

## 2.Installation and configuration

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```
PS C:\Users\pierre> python
Python 3.7.1 (default, Dec 10 2018, 22:54:23) [MSC v.1915 64 bit (AMD64)]
Type "help", "copyright", "credits" or "license" for more information
>>>
```


## With MAC-OS

```
iMac-de-pierre:Downloads$ python
Python 3.7.1 (default, Dec 14 2018, 19:28:38)
[Clang 4.0.1 (tags/RELEASE_401/final)] :: Anaconda, Inc. on darwin
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>>>
```


## With LINUX

## lama@lama-pc: ~

-(ㅁ)

```
File Edit View Search Terminal Help
```

(base) lama@lama-pc:~\$ python3
Python 3.8.3 (default, Jul 2 2020, 16:21:59)
[GCC 7.3.0] :: Anaconda, Inc. on linux
Type "help", "copyright", "credits" or "license" for more information.

## Notations

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## Python

```
>>> print('Hello World!')
```

Hello World!

```
>>> 2 + 5
```

7
>>> print('Welcome to Real Python!')
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## Spyder



## Spyder

## Save file



## Spyder

## Spyder (Python 3.8)

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File Edit Search Source Run Debug Consoles Projects Tools View Help
```



三
\#!/usr/bin/env python3
\#!/usr/bin/env python3
\# - *- coding: utf-8 -*.
\# - *- coding: utf-8 -*.
Created on Thu Sep 10 10:37:11 2020
Created on Thu Sep 10 10:37:11 2020
@author: lana
@author: lana
print ('Welcome to the CS class 202d')
print ('Welcome to the CS class 202d')

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## Spyder

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3 "n"
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s
9 print('Welcome to the CS closs 2020')

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## Jupyter



## Jupyter

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## In [ ]: print("Welcome Class 2020"

## Jupyter

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Welcome Class 2020
In [ ]:

## Embeded Animation



Jupyter Notebooks via https://jupyter.org/

## Outline

## Session II

(1) Bugs
(2) Types
(3) Built-in functions

- Operations
- Simple examples


## 1.Bugs

## History

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## Remark

In 1946, when Hopper was released from active duty, she joined the Harvard Faculty at the Computation Laboratory where she continued her work on the Mark II and Mark III. Operators traced an error in the Mark II to a moth trapped in a relay, coining the term bug. This bug was carefully removed and taped to the log book. Stemming from the first bug, today we call errors or glitches in a program a bug.


Emperor gum moth, Opodiphthera
eucalypti

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## Bugs

As defined in Wikipedia:

## Definition

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This helps for faster reaction and most importantly, appropriate reaction.

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(0) Missing command Errors: When an expected command is missing.
(0) Syntactic Error: They are misspelled words or grammatically incorrect sentences.

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In Python, the most famous Bugs are:

- Syntax: bugs in structure of the input;
- Routine: bugs in execution;
- Semantic: the program is correct in syntax and execution, but the output is not we expected.


## Examples

```
>>> 10 * (1/0)
Traceback (most recent call last):
    File "<stdin>", line 1, in <module>
ZeroDivisionError: division by zero
>>> 4 + spam*3
Traceback (most recent call last):
    File "<stdin>", line 1, in <module>
NameError: name 'spam' is not defined
>>> '2' + 2
Traceback (most recent call last):
    File "<stdin>", line 1, in <module>
TypeError: Can't convert 'int' object to str implicitly
```


## 2.Types

## Basic Data Types in Python

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There are several basic numeric, string, and Boolean types that are built into Python.

We will see some Python's built-in functions like the built-in print()function.

## Integers

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Python
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```

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>>> print(10)
10

## Floating-Point Numbers

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## Python

```
>>> 4.2
4.2
>>> type(4.2)
<class 'float'>
>>> 4.
4.0
>>> .2
0.2
>>> .4e7
4000000.0
>>> type(.4e7)
<class 'float'>
>>> 4.2e-4
0.00042
```


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```
>>> 1.79e308
1.79e+308
>>> 1.8e308
inf
```


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## Python

>>> 5e-324
$5 e-324$
>>> 1e-325
0.0

## Complex Numbers

Complex numbers are specified as $<$ real part $>+<$ imaginary part $>\mathbf{j}$.

## Python

>>> $2+3 j$
(2+3j)
>>> type(2+3j)
<class 'complex'>

## Boolean Type, Boolean Context, and "Truthiness".

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Python 3 provides a Boolean data type. Objects of Boolean type may have one of two values, True or False:
(1) $a==b$ : $a$ is equal to $b$,
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Note: expressions $=>$ and $=<$ do not exist in Python.

Try: $1==2 ; 1>3 ; 5!=3 ; x=2 ; x+4>=1$.

## IF TEST

## Example

If $\mathrm{a}==\mathrm{b}$ :
print('The two numbers are equal')
else:
print('The two numbers are different')

## Example

If $a==b$ :
print('The two numbers are equal')
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## Example

def testequal $(a, b)$ :
If $a==b$ :
print('The two numbers are equal')
return(2*a)
else:
print('The two numbers are different')
return $(a+b)$

## Outline

## Session III

© Recall+Exercise
(2) Built-in functions

- Operations
- Conditional execution
- Simple examples


## Exercise

Write a function that returns the factorial of a number " n ".

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Hint
$n!=(n-1)$ !. $n$ if $n \neq 0$,
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Write a function that returns the factorial of a number " $n$ ".
Hint
$n!=(n-1)!. n$ if $n \neq 0$,
$0!=1$

## Solution

```
def fact(n):
            if n!=0:
                return(fact(n-1)*n)
    else:
                                return(1)
    fact(5)
120
    |
```


## Exercise

Write a function that returns the factorial of a number " n ".

## Hint

$n!=(n-1)!. n$ if $n \neq 0$,
$0!=1$

## Solution



What happens if we test fact(-2)? Find a solution. fact(0.5)?

## Strings

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(2) String literals are delimited using either single quote ' ' or double quotes " ". All the characters between the opening and closing delimiter are part of the string:

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```
Python
>>> print("I am a string.")
I am a string.
>>> type("I am a string.")
<class 'str'>
>>> print('I am too.')
I am too.
>>> type('I am too.')
<class 'str'>
```


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11

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>>> print('This string contains a single quote (') character.')
SyntaxError: invalid syntax

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## Python

```
>>> print("This string contains a single quote (') character.")
This string contains a single quote (') character.
>>> print('This string contains a double quote (") character.')
This string contains a double quote (") character.
```


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(0) A string in Python can contain as many characters as you wish. It can also be empty:
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## Python

>>> print('This string contains a single quote ( $\backslash$ ') character.')
This string contains a single quote (') character.

## Strings

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## OR

```
Python
>>> print("This string contains a double quote (\") character.")
This string contains a double quote (") character.
```


## Escape sequences

Escape Usual Interpretation of

Sequence Character(s) After Backslash
"Escaped" Interpretation

| $\^{\prime}$ | Terminates string with single quote opening <br> delimiter |
| :--- | :--- |
| $\^{\prime \prime} \quad$ | Terminates string with double quote <br> opening delimiter |

Literal single quote (') character

Literal double quote (") character
\newline Terminates input line
Newline is ignored

Introduces escape sequence
Literal backslash ( $\backslash$ )
character

## 2.Built-in functions

## Print()-function

- Pressing Enter in the middle of a string will cause Python to think it is incomplete:


## Python

>>> print('a

SyntaxError: EOL while scanning string literal

## Print()-function

- Pressing Enter in the middle of a string will cause Python to think it is incomplete:
- To break up a string over more than one line:


## Python

```
>>> print('a\
... b\
... c')
```

abc

## Print()-function

- Pressing Enter in the middle of a string will cause Python to think it is incomplete:
- To break up a string over more than one line:
- To include a literal backslash in a string:


## Python

>>> print('foo<br>bar')
foo\bar

## Print()-function

- Pressing Enter in the middle of a string will cause Python to think it is incomplete:
- To break up a string over more than one line:
- To include a literal backslash in a string:
- A tab character can be specified by the escape sequence " $\backslash t$ "

```
Python
>>> print('foo\tbar')
foo bar
```


## Print()-function

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- To break up a string over more than one line:
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- Some examples

```
Python
>>> print("a\tb")
a b
>>> print("a\141\x61")
aaa
>>> print("a\nb")
a
b
>>> print('\u2192 \N{rightwards arrow}')
```


## Print()-function

- Pressing Enter in the middle of a string will cause Python to think it is incomplete:
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- To include a literal backslash in a string:
- A tab character can be specified by the escape sequence " $\backslash t$ "
- Some examples
- A raw string literal is preceded by $r$ or $R$, which specifies that escape sequences in the associated string are not translated.


## Python

```
>>> print('foo\nbar')
foo
bar
>>> print(r'foo\nbar')
foo\nbar
>>> print('foo\\bar')
foo\bar
>>> print(R'foo\\bar')
foo\\bar
```


## Print()-function

- Pressing Enter in the middle of a string will cause Python to think it is incomplete:
- To break up a string over more than one line:
- To include a literal backslash in a string:
- A tab character can be specified by the escape sequence " $\backslash t$ "
- Some examples
- A raw string literal is preceded by $r$ or $R$, which specifies that escape sequences in the associated string are not translated.
- Triple-Quoted Strings provides a convenient way to create a string with both single and double quotes in it.

```
Python
>>> print('''This string has a single (') and a double (") quote.''')
This string has a single (') and a double (') quote.
```


## Built-in functions

```
Python
```

```
>>> type(True)
```

>>> type(True)
<class 'bool'>
<class 'bool'>
>>> type(False)
>>> type(False)
<class 'bool'>

```
<class 'bool'>
```


## Built-in functions

## Math \#

## Function Description

abs() Returns absolute value of a number
divmod() Returns quotient and remainder of integer division
$\max () \quad$ Returns the largest of the given arguments or items in an iterable
min() Returns the smallest of the given arguments or items in an iterable
pow() Raises a number to a power
round () Rounds a floating-point value
sum ( ) Sums the items of an iterable

## Conversion of types

The duty of the functions $\operatorname{int}()$, float(), and $\operatorname{str}()$ is to convert the type. To pass from integer to string or float and vice-versa.

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## lama@lama-pc: ~

```
File Edit View Search Terminal Help
```

(base) lama@lama-pc:~\$ python3
Python 3.8.3 (default, Jul 2 2020, 16:21:59)
[GCC 7.3.0] :: Anaconda, Inc. on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> $i=3$
>>> str(i)
'3'
>>> $i=$ '456'
>>> int(i)
456
>>> float(i)
456.0
>>> $i=$ '3.1416'
>>> float(i)
3.1416

## 3.Operations

## Numerical operations

Using Python shell as a calculator : introducing in Python basic mathematical operations.
(1) A-S Addiction $a+b$ and Subtraction $a-b$;

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Left-to-right order: $F \sim R \sim M \sim D$.

## Example

Predict the answer in Python: $2+3 ; 2 *(3-1) ; 2 * 3-1 ;(1+1) * *(5-2)$; $3 * 1 * * 3 ; 2 / 3 ; 2 / 0 ;(3 * 1) * * 3 ; 4 / / 3 ; 2 / / 3 ; 17 \% 3$ and $15 \% 4$.

## Example

Predict the answer in Python: $2+3 ; 2 *(3-1) ; 2 * 3-1 ;(1+1) * *(5-2)$; $3 * 1 * * 3 ; 2 / 3 ; 2 / 0 ;(3 * 1) * * 3 ; 4 / / 3 ; 2 / / 3 ; 17 \% 3$ and $15 \% 4$.

```
>>> 2+3
5
```

```
>>> 2*3-1
```

>>> 2*3-1
5

```
```

>>> 3*1**3
3

```
>>> (3*1)**3
27
>>> 4//3
1
```

>>> 2//3

```
0
```

>>> 2*(3-1)
4
>>> (1+1)**(5-2)
8
>>> 2/3
0.6666666666666666

```
```

>>> 2/0

```
Traceback (most recent call last):
File "<pyshell\#3>", line 1, in <module>
2/0
ZeroDivisionError: int division or modu
```

>>> 17%3
2
>>> 15%4
3

```

\section*{Operations on Strings}
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>>> 'hello'*3
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>>> 'hello' + 'world'
'helloworld'

```

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(1) \(*\) is used for the repetition;
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```

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'hellohellohello'
>>> 'hello' + 'world'
'helloworld'

```
© \# is used to insert comments in the text.
```

>>> 'hello' + 'world' \# concatenation of strings
'helloworld'

```

\section*{Exercise I}

Predict the answer of the following expressions, then execute it on Python.
\(-(1+2) * * 3\)
- "Da" * 4
— "Da" + 3
— ("Pa"+"La") * 2
— ("Da"*4) / 2
- 5 / 2
\(-5 / / 2\)
\(-5 \% 2\)
— str(4) * int("3")
— int("3") + float("3.2")
- str(3) * float("3.2")
\(-\operatorname{str}(3 / 4)\) * 2

\section*{Exercise II-III}

Exercise 2: What is the volume of a sphere with radius 5 ?

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```

r = 5 \#sphere radious
print(4/3 *3.14 *(r**3)) \#shpere volume

```
>>>
523.333333333

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```

\section*{>>>}

\subsection*{523.333333333}

Exercise 3: Suppose the cover price of a book is 24.95 \$ but book stores get a \(\mathbf{4 0 \%}\) discount. Shipping costs \(3 \mathbb{\$}\) for the first copy and 75 cents for each additional copy. What is the total wholesale cost for 60 copies?

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```

p = 24.95 \#original price
s = p*60/100 \#discount price
t1 = s+3 \#first book delivered
t2 = s+0.75 \#other books delivered
print(t1 + 59*t2) \#final price

```
```

>>>
945.45

```

\section*{Outline}

\section*{Session IV}
(1) Modulus
(2) Lists
(3) Operations on Lists

\section*{1.Modulus}

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(1) Open the module with command import;

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If we run the name of the modulus we can learn if it is already in our version of Python.

To use one of the function in the modulus:
(1) Open the module with command import;
(2) Specify the name of the module and the name of the function, separated by a dot (dot notation).

\section*{Math Module}
```

>>> math
<module 'math' (built-in)>
>>> import math
>>>
>>> import math
>>> math.pi
3.141592653589793
>>> import math
>>> math.e
2.718281828459045

```

\section*{Math Module}
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<module 'math' (built-in)>
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Module name can be reassigned using command as.

\section*{Math Module}
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>>> math.pi
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>>> import math
>>> math.e
2.718281828459045

```

Module name can be reassigned using command as.
```

>>> import math as mt
>>> mt.e
2.718281828459045

```
```

>>>mt.sin(0)

```
>>>mt.sin(0)
>>> 0.0
>>> mt.sin(90)
>>> 0.8939966636005579
>>> degrees = 90
>>> radians = degrees / 360.0 * 2 * math.pi
>>> math.sin(rad)
1.0
```


## Exercises

1-Calculate the sine of the angle $\frac{3 \pi}{4}$.

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## lama@lama-pc: ~

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File Edit View Search Terminal Help
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Python 3.8.3 (default, Jul 2 2020, 16:21:59)
[GCC 7.3.0] :: Anaconda, Inc. on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> import math
>>> math.sin(3*math.pi/4)
0.7071067811865476
>>>
```

2-Calculate the length of the hypotenuse of a right triangle having 9 and 12 as dimensions for the other two sides.

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>>> math.sin(3*math.pi/4)
0.7071067811865476
>>>
```

2-Calculate the length of the hypotenuse of a right triangle having 9 and 12 as dimensions for the other two sides.

```
>>> def hypo(a,b):
    h=a*a+b*b
    s=math.sqrt(h)
... s=math.sqre(h)
... return(s)
>>> hypo(3,4)
5.0
>>> hypo(9,12)
15.0
```


## Random Module

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Return the next random floating point number in the range $[0.0,1.0)$.

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Return a random floating point number $N$ such that $a<=N<=b$ for $a<=b$ and $b<=N<=a$ for $b<a$.

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Return a random element from the non-empty sequence seq. If seq is empty, raises IndexError.

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- random.shuffle(seq)

Returns a shuffeled sequence.

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- random.choice(seq)

Return a random element from the non-empty sequence seq. If seq is empty, raises IndexError.

- random.shuffle(seq)

Returns a shuffeled sequence.

- random.sample(seq, $\mathbf{k}$ )

Return a $k$ length list of unique elements chosen from the sequence.

## Examples

>>> random()
0.37444887175646646
\# Random float: $0.0<=x<1.0$
\# Random float: $2.5<=x<10.0$
\# Interval between arrivals averag
\# Integer from 0 to 9 inclusive
\# Even integer from 0 to 100 inclus
\# Single random element from a seq \# Shuffle a list
>>> deck = ace the shuffle(deck)
>> deck
['four', 'two', 'ace', 'three']
>>> sample([10, 20, 30, 40, 50], k=4) \# Four samples without replacement

## Exercise

Write a Python programm allowing us to get the roots of a second degree equation when they exist.

## Exercise

## Write a Python programm allowing us to get the roots of a second degree equation when they exist.

```
In [5]: import math
def roots(a,b,c):
    d=b**2-4*a*c
    if d>0:
        r=math.sqrt(d)
        x1=(-b-r)/(2*a)
        x2=(-b+r)/(2*a)
        return('There exists two distinct solutions:',x1,x2)
    if d==0:
        r=math.sqrt(d)
        x=-b/(2*a)
        return('There exists one double solution:',x)
    else:
            return('the equation has no solution in R2')
In [6]: roots(1,2,1)
Out[6]: ('There exists one double solution:', -1.0)
In [7]: roots(1,-4,5)
Out[7]: 'the equation has no solution in R2'
In [8]: roots(1,5,6)
Out[8]: ('There exists two distinct solutions:', -3.0, -2.0)
```


## 2.Lists

## Lists in Python


(1) A list is a data structure in Python that is a mutable, or changeable, ordered sequence of elements.

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(2) Each element or value that is inside of a list is called an item.

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(1) A list is a data structure in Python that is a mutable, or changeable, ordered sequence of elements.
(3) Each element or value that is inside of a list is called an item.
(0) Just as strings are defined as characters between quotes, lists are defined by having values between square brackets [ ].
(1) Lists are great to use when you want to work with many related values.

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- They enable you to keep data together that belongs together, condense your code, and perform the same methods and operations on multiple values at once.
(1) Lists are great to use when you want to work with many related values.
( They enable you to keep data together that belongs together, condense your code, and perform the same methods and operations on multiple values at once.
- When thinking about Python lists and other data structures that are types of collections, it is useful to consider all the different collections you have on your computer: your assortment of files, your song playlists, your browser bookmarks, your emails, the collection of videos you can access on a streaming service, and more.

To get started, let's create a list that contains items of the string data type:

```
sea_creatures = ['shark', 'cuttlefish', 'squid', 'mantis shrimp', 'anemone']
```

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When we print out the list, the output looks exactly like the list we created:

```
print(sea_creatures)
Output
['shark', 'cuttlefish', 'squid', 'mantis shrimp', 'anemone']
```

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When we print out the list, the output looks exactly like the list we created:

```
print(sea_creatures)
```

Output
['shark', 'cuttlefish', 'squid', 'mantis shrimp', 'anemone']

As an ordered sequence of elements, each item in a list can be called individually, through indexing.

## Indexing lists

Each item in a list corresponds to an index number, which is an integer value, starting with the index number 0 .

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Each item in a list corresponds to an index number, which is an integer value, starting with the index number 0 .

| 'shark' | 'cuttlefish' | 'squid' | 'mantis shrimp' | 'anemone' |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 2 | 3 | 4 |

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Each item in a list corresponds to an index number, which is an integer value, starting with the index number 0 .

| 'shark' | 'cuttlefish' | 'squid' | 'mantis shrimp' | 'anemone' |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 2 | 3 | 4 |

Because each item in a Python list has a corresponding index number, we're able to access and manipulate lists in the same ways we can with other sequential data types.

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Each item in a list corresponds to an index number, which is an integer value, starting with the index number 0 .

| 'shark' | 'cuttlefish' | 'squid' | 'mantis shrimp' | 'anemone' |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 2 | 3 | 4 |

Because each item in a Python list has a corresponding index number, we're able to access and manipulate lists in the same ways we can with other sequential data types.

```
sea_creatures[0] = 'shark'
sea_creatures[1] = 'cuttlefish'
sea_creatures[2] = 'squid'
sea_creatures[3] = 'mantis shrimp'
sea_creatures[4] = 'anemone'
```


## Negative indexing

NB: If we call the list with an index numberthat is greater than 4 , it will be out of range as it will not be valid:

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NB: If we call the list with an index numberthat is greater than 4 , it will be out of range as it will not be valid:

```
print(sea_creatures[18])
```

```
Output
IndexError: list index out of range
```


## Negative indexing

NB: If we call the list with an index numberthat is greater than 4 , it will be out of range as it will not be valid:

```
print(sea_creatures[18])
```


## Output

```
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In addition to positive index numbers, we can also access items from the list with a negative index number, by counting backwards from the end of the list, starting at -1 .

## Negative indexing

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```
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```


## Output

## IndexError: list index out of range

In addition to positive index numbers, we can also access items from the list with a negative index number, by counting backwards from the end of the list, starting at -1 .

| 'shark' | 'cuttlefish' | 'squid' | 'mantis shrimp' | 'anemone' |
| :---: | :---: | :---: | :---: | :---: |
| -5 | -4 | -3 | -2 | -1 |

## Negative indexing

NB: If we call the list with an index numberthat is greater than 4 , it will be out of range as it will not be valid:

```
print(sea_creatures[18])
```


## Output

```
IndexError: list index out of range
```

In addition to positive index numbers, we can also access items from the list with a negative index number, by counting backwards from the end of the list, starting at -1 .
print(sea_creatures[-3])

```
Output
squid
```


## Modifying Lists

If we want to change the string value of the item at index 1 from 'cuttlefish' to 'octopus', we can do so like this:

## Modifying Lists

If we want to change the string value of the item at index 1 from 'cuttlefish' to 'octopus', we can do so like this:

```
sea_creatures[1] = 'octopus'
```

Now when we print sea_creatures, the list will be different:

```
print(sea_creatures)
```

Output
['shark', 'octopus', 'squid', 'mantis shrimp', 'anemone']

## Modifying Lists

If we want to change the string value of the item at index 1 from 'cuttlefish' to 'octopus', we can do so like this:
We can also change the value of an item by using a negative index number instead:

## Modifying Lists

If we want to change the string value of the item at index 1 from 'cuttlefish' to 'octopus', we can do so like this:
We can also change the value of an item by using a negative index number instead:

```
sea_creatures[-3] = 'blobfish'
print(sea_creatures)
```

Output
['shark', 'octopus', 'blobfish', 'mantis shrimp', 'anemone']

## Slicing lists

We can also call out a few items from the list.
Let's say we would like to just print the middle items of the list, we can do so by creating a slice. With slices, we can call multiple values by creating a range of index numbers separated by a colon $[x: y]$ :

## Slicing lists

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```
print(sea_creatures[1:4])
```

Output
['octopus', 'blobfish', 'mantis shrimp']

## Slicing lists

We can also call out a few items from the list.
Let's say we would like to just print the first items of the list till $n^{\text {th }}$ item of the list, we can do so by creating a slice. With slices, we can call multiple values by creating a range of index numbers separated by a colon [: n]:

```
print(sea_creatures[:3])
```

Output
['shark', 'octopus', 'blobfish']

## Slicing lists

We can also call out a few items from the list.
Let's say we would like to just print the items from $n$ till the end of the list, we can do so by creating a slice. With slices, we can call multiple values by creating a range of index numbers separated by a colon [ $n$ :]:

```
print(sea_creatures[2:])
```

Output
['blobfish', 'mantis shrimp', 'anemone']

## Slicing lists

We can also call out a few items from the list.
Let's say we would like to just print the middle items with negative index of the list, we can do so by creating a slice. With slices, we can call multiple values by creating a range of index numbers separated by a colon $[-x:-y]$ :

```
print(sea_creatures[-4:-2])
print(sea_creatures[-3:])
```

```
Output
['octopus', 'blobfish']
['blobfish', 'mantis shrimp', 'anemone']
```

One last parameter that we can use with slicing is called stride, which refers to how many items to move forward after the first item is retrieved from the list.

One last parameter that we can use with slicing is called stride, which refers to how many items to move forward after the first item is retrieved from the list. The syntax for this construction is list $[x: y: z]$, with $z$ referring to stride.

```
numbers = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]
print(numbers[1:11:2])
```

Output
$[1,3,5,7,9]$

One last parameter that we can use with slicing is called stride, which refers to how many items to move forward after the first item is retrieved from the list. We can omit the first two parameters and use stride alone as a parameter with the syntax list[:: z]:

## print(numbers[::3])

## Output

$$
[0,3,6,9,12]
$$

## 1.Operations on Lists

## Operations on Lists

1) Adding two lists using + to concatenate them:
$l_{1}+I_{2}=l_{3}$
```
sea_creatures = ['shark', 'octopus', 'blobfish', 'mantis shrimp', 'anemone']
oceans = ['Pacific', 'Atlantic', 'Indian', 'Southern', 'Arctic']
print(sea_creatures + oceans)
```


## Operations on Lists

1) Adding two lists using + to concatenate them:
2) Extending a list with another list using extend():
$l_{1} \cdot \operatorname{extend}\left(l_{2}\right)=l_{1} ; l_{1}=l_{1}+l_{2}$
```
    In [9]: l1=[1,2,3,4,5,6]
    l2=[11,12]
In [10]: l1.extend(l2)
In [11]: l1
Out[11]: [1, 2, 3, 4, 5, 6, 11, 12]
```


## Operations on Lists

1) Adding two lists using + to concatenate them:
2) Extending a list with another list using extend():
3) Adding an item to the list using append():
$I_{1} \cdot \operatorname{append}(a)=I_{1}$;
>>> stack $=[3,4,5]$
>>> stack.append(6)
>>> stack.append(7)
>>> stack
[3, 4, 5, 6, 7]

## Deletion

We must distinguish between two ideas, revoming and deleting :

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```
In [12]: l1
Out[12]: [1, 2, 3, 4, 5, 6, 11, 12]
In [13]: l1.remove(5)
In [14]: l1
Out[14]: [1, 2, 3, 4, 6, 11, 12]
```


## Deletion

We must distinguish between two ideas, revoming and deleting : A specific item a can be removed from a list "l" as follows: An item at position $n$ can be deleted from a list " $l$ " as follows:

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We must distinguish between two ideas, revoming and deleting : A specific item a can be removed from a list "l" as follows: An item at position $n$ can be deleted from a list " $l$ " as follows:

```
# list of numbers
n_list = [1, 2, 3, 4, 5, 6]
# Deleting 2nd element
del n_list[1]
```


## Deletion

We must distinguish between two ideas, revoming and deleting : A specific item a can be removed from a list "l" as follows: An item at position $n$ can be deleted from a list " 1 " as follows:
We can remove all the items from the list by using:

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```
In [19]: l1=[1, 2, 3, 4, 5, 6, 11, 12]
In [20]: l1.pop()
Out[20]: 12
```


## More operations

We can ask for the length of a list / by using:

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We can ask for the length of a list / by using:

```
In [24]: l1
Out[24]: [1, 2, 3, 4, 5, 6, 11]
In [25]: len(l1)
Out[25]: 7
```


## More operations

We can ask for the length of a list / by using: We can count the occurences of a certain element $a$ in I by using:

## More operations

We can ask for the length of a list / by using:
We can count the occurences of a certain element $a$ in / by using:
In [29]: 12
Out [29]: $[3,2,4,3,3,2,4,5]$
In [30]: $12 . \operatorname{count}(3)$
Out [30]: 3

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We can ask for the length of a list / by using: We can count the occurences of a certain element $a$ in / by using: We can ask for the index of an item in a list / by using:

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We can ask for the length of a list / by using:
We can count the occurences of a certain element $a$ in $/$ by using: We can ask for the index of an item in a list / by using:

```
In [32]: l1
Out[32]: [1, 2, 3, 4, 5, 6, 11]
In [33]: ll.index(3)
Out[33]: 2
```


## More operations

We can ask for the length of a list / by using: We can count the occurences of a certain element $a$ in I by using: We can ask for the index of an item in a list / by using: Note that: If a list contains the same element several times, we can ask for the index starting a certain position :

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We can count the occurences of a certain element $a$ in / by using:
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Note that: If a list contains the same element several times, we can ask for the index starting a certain position :

```
In [50]: l2=[3,2,4,3,3,2,4,5]
In [51]: l2.index(3,2)
Out[51]: 3
```


## More operations

We can ask for the length of a list / by using:
We can count the occurences of a certain element $a$ in I by using:
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Note that: If a list contains the same element several times, we can ask for the index starting a certain position :
We can ask for the reverse of a list / by using:

```
In [38]: l1
Out[38]: [1, 2, 3, 4, 5, 6, 11]
In [39]: l1.reverse()
In [40]: l1
Out[40]: [11, 6, 5, 4, 3, 2, 1]
```


## More operations

We can ask for the length of a list / by using:
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Note that: If a list contains the same element several times, we can ask for the index starting a certain position :
We can ask for the reverse of a list / by using:
We can get a copy of the list / by using:
In [42]: 11
Out [42]: $[11,6,5,4,3,2,1]$
In [43]: $12=11 . \operatorname{copy}()$
In [44]: 12
Out [44]: $[11,6,5,4,3,2,1]$

## More operations

We can ask for the length of a list / by using:
We can count the occurences of a certain element $a$ in I by using:
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We can sort a list I in a decreasing or increasing order as follows: (PS, by default it is an increasing order)

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We can ask for the reverse of a list / by using:
We can get a copy of the list / by using:
We can sort a list I in a decreasing or increasing order as follows: (PS, by default it is an increasing order)

```
In [46]: l2
Out[46]: [1, 2, 3, 4, 5, 6, 11]
In [48]: l2.sort(reverse=True)
In [49]: l2
Out[49]: [11, 6, 5, 4, 3, 2, 1]
```


## Exercise

Using the operations on lists to write a function that returns the min of a list I.

## Exercise

Using the operations on lists to write a function that returns the min of a list l.

# In [58]: def minlist(l): <br> l.sort() <br> l. reverse() <br> a=l.pop() <br> return a <br> In [53]: $l=[4,6,1,7,8,4,3,9,0]$ <br> In [59]: minlist(l) <br> Out[59]: 0 

## Exercise 2

Define the following list: $L=[17,38,10,25,72]$ then do the following:
(1) sort and show the list.
(3) add number 12 to the list.
(3) Give the reverse of the list.
(1) Give the index of the elemnt 17 .
(0) Remove the element 38 from the list.

- Show the sub-list $Q$ composed of the second to the third element of $L$.
( Ohow the sub-list $R$ composed of the elements of $L$ from the beginning to the second element.
(3) Show the sub-list $T$ composed of the elements of $L$ from its third element till the end.
- Show the sub-list $C$ composed of all the elements of $L$.
(10) Show the last and the third element of $L$ using negative indexing.


## Solutions

```
In [1]: L=[17, 38,10, 25,72]
In [2]: L.sort()
In [3]: L
Out[3]: [10, 17, 25, 38, 72]
In [4]: L.append(12)
In [5]: L
Out[5]: [10, 17, 25, 38, 72, 12]
In [6]: L.reverse()
In [7]: L
Out[7]: [12, 72, 38, 25, 17, 10]
In [8]: L.index(17)
Out[8]: 4
In [9]: L.remove(38)
In [10]: L
Out[10]: [12, 72, 25, 17, 10]
In [14]: Q=L[1:3]
In [15]: Q
Out[15]: [72, 25]
In [16]: R=L[:2]
In [17]: R
Out[17]: [12, 72]

\section*{Solutions}
```

In [18]: T=L[2:]
In [19]: T
Out[19]: [25, 17, 10]
In [20]: C=L.copy()
In [21]: C
Out[21]: [12, 72, 25, 17, 10]
In [22]: L[-1]
l=len(L)
L[-l+2]
Out[22]: 25
In [23]: l
Out[23]: 5
In [24]: L[-l+2]
Out[24]: 25
In [25]: L[-1]
Out[25]: 10

```

\section*{Range() and List() Functions}

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\[
\begin{aligned}
& \mid \ggg>\text { list (range }(10)) \\
& 2 \mid[0,1,2,3,4,5,6,7,8,9]
\end{aligned}
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It is possible to give two or three arguments to the list function as shown:

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& \text { 1| } \ggg>\text { list (range (10)) } \\
& 2 \mid[0,1,2,3,4,5,6,7,8,9]
\end{aligned}
\]

It is possible to give two or three arguments to the list function as shown:
```

>>> list(range(0, 5))
[0, 1, 2, 3, 4]
>>> list(range(15, 20))
[15, 16, 17, 18, 19]
>>> list(range(0, 1000, 200))
[0, 200, 400, 600, 800]
>>> list(range(2, -2, -1))
[2, 1, 0, -1]

```

\section*{Range() and List() Functions}

The instruction range() is a special function in Python that generates integer numbers included in a certain interval.
When it is combined with the list() function, it gives a list of integers.
```

1| >>> list (range(10))
2| [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

```

It is possible to give two or three arguments to the list function as shown:
\[
\begin{aligned}
& \text { [] } \ggg \text { list }(\text { range }(10,0)) \\
& \text { [ } 10
\end{aligned}
\]

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\begin{aligned}
& \mid \ggg>\text { list (range }(10)) \\
& {[0,1,2,3,4,5,6,7,8,9]}
\end{aligned}
\]

It is possible to give two or three arguments to the list function as shown:
\[
\begin{aligned}
& \mid \ggg>\text { list }(\text { range }(10,0,-1)) \\
& 2 \mid[10,9,8,7,6,5,4,3,2,1]
\end{aligned}
\]

\section*{List of lists}

This functionality called sometimes Matrix format is pratical in several ways.

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```

In [26]: L1=['lion',4]
L2=['tigre',3]
L3=['zebra',5]
Z=[L1,L2,L3]
In [27]: Z
Out[27]: [['lion', 4], ['tigre', 3], ['zebra', 5]]
In [28]: Z[0]
Out[28]: ['lion', 4]
In [29]: Z[1]
Out[29]: ['tigre', 3]
In [30]: Z[2]((-%5Cbigcirc=))
Out[30]: ['zebra', 5]

```

\section*{List of lists}

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```

In [26]: L1=['lion',4]
L2=['tigre',3]
L3=['zebra',5]
Z=[L1,L2,L3]
In [27]: Z
Out[27]: [['lion', 4], ['tigre', 3], ['zebra', 5]]
In [28]: Z[0]
Out[28]: ['lion', 4]
In [29]: Z[1]
Out[29]: ['tigre', 3]
In [30]: Z[2]((-%5Cbigcirc=))
Out[30]: ['zebra', 5]

```

In order to reach an element of the sub-list, we need a double indexing.

\section*{List of lists}

This functionality called sometimes Matrix format is pratical in several ways.
```

In [26]: L1=['lion',4]
L2=['tigre',3]
L3=['zebra',5]
Z=[L1,L2,L3]
In [27]: Z
Out[27]: [['lion', 4], ['tigre', 3], ['zebra', 5]]
In [28]: Z[0]
Out[28]: ['lion', 4]
In [29]: Z[1]
Out[29]: ['tigre', 3]
In [30]: Z[2]((-%5Cbigcirc=))
Out[30]: ['zebra', 5]

```

In order to reach an element of the sub-list, we need a double indexing.
```

In [31]: Z[2][0]
Out[31]: 'zebra'
In [32]: Z[1][1]
Out[32]: 3

```

\section*{Exercise}

Create 4 lists called Autumn, Winter, Spring and Summer that contain each all the corresponding months. Create a list \(S\) that contains the four previous lists. Predict then verify the output of each expression:
\[
S[2], S[1][0], S[1: 2] \text { and } S[:][1] .
\]
```

In [33]: Autumn=['september','october','november']
Winter=['december','january','february']
Spring=['march','april','may']
Summer=['june','july','august']
S=[Autumn, Winter,Spring,Summer]
In [34]: S
Out[34]: [['september', 'october', 'november'],
['december', 'january', 'february'],
['march', 'april', 'may'],
['june', 'july', 'august']]

```
```

In [35]: S[2]((-%5Cbigcirc=))

```
In [35]: S[2]
Out[35]: ['march', 'april', 'may']
In [36]: S[1][0]
Out[36]: 'december'
In [37]: S[1:2]
Out[37]: [['december', 'january', 'february']]
In [38]: S[:][1]
Out[38]: ['december', 'january', 'february']
```


## 2.Loops and comparision

## Loop FOR

In programming, we mostly need to repeat an instruction several times. That's why we need LOOPS.

## Loop FOR

In programming, we mostly need to repeat an instruction several times. That's why we need LOOPS.

```
In [40]: M=['lion','tigre', 'monkey', 'zebra', 'bear', 'snake']
In [43]: print(M[0])
    lion
In [44]: print(M[1])
In [45]: print(M[2])
    monkey
In [46]: }\mp@subsup{\underbrace}{\mathrm{ zebra }}{\operatorname{print(M[3])}
In [47]: for i in M:
    print(i)
lion
tigre
monkey
zebra
bear
snake
```

- The letter $i$ in the loop for is called ITERATION variable that changes its value at each iteration of the loop.
- The letter $i$ in the loop for is called ITERATION variable that changes its value at each iteration of the loop.
- It is a dummy variable, which means can be replaced by anything else.

```
In [48]: for animal in M:
    print(animal)
lion
tigre
monkey
zebra
bear
snake
```

- The letter $i$ in the loop for is called ITERATION variable that changes its value at each iteration of the loop.
- It is a dummy variable, which means can be replaced by anything else.
- The iteration variable can be of any type depending on the list.
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- The letter $i$ in the loop for is called ITERATION variable that changes its value at each iteration of the loop.
- It is a dummy variable, which means can be replaced by anything else.
- The iteration variable can be of any type depending on the list.
- We caracterize it by the $<:>$ at the end of the line. This means that the loop for is waiting for a BLOC of instructions.
- This bloc is considered as the body of the loop.
- The letter $i$ in the loop for is called ITERATION variable that changes its value at each iteration of the loop.
- It is a dummy variable, which means can be replaced by anything else.
- The iteration variable can be of any type depending on the list.
- We caracterize it by the $<:>$ at the end of the line. This means that the loop for is waiting for a BLOC of instructions.
- This bloc is considered as the body of the loop.
- In order to know where the bloc starts or ends, we use the identation which is made of the 4 spaces (1 TAB) with respect to the position of the word FOR.,


## Example

```
In [49]: for animal in M : print ( animal )
    print ( animal *2)
print (" it is over ")
```


## Example

```
In [49]: for animal in M : print ( animal )
    print ( animal *2)
print (" it is over ")
```

lion
lionlion
tigre
tigretigre
monkey
monkeymonkey
zebra
zebrazebra
bear
bearbear
snake
snakesnake
it is over

The iteration can be over different types of lists and sub-lists:

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In [50]: for animal in M[1:3]:
print(animal)
tigre
monkey

The iteration can be over different types of lists and sub-lists:

```
In [50]: for animal in M[1:3]:
    print(animal)
```

```
In [51]: for i in [1,2,3]:
    print(i)
```

tigre
1
monkey
2

The iteration can be over different types of lists and sub-lists:

```
In [50]: for animal in M[1:3]:
    print(animal)
```

```
In [51]: for i in [1,2,3]:
    print(i)
```

```
```

In [52]: for i in range(4):

```
```

In [52]: for i in range(4):
print(i)

```
```

    print(i)
    ```
```

    0
    1
2
3

The iteration can be over different types of lists and sub-lists:

```
In [50]: for animal in M[1:3]:
    print(animal)
```

```
In [51]: for i in [1,2,3]:
    print(i)
1
```

tigre monkey

```
In [52]: for i in range(4):
    print(i)
0
1
2
3
```

```
In [53]: for i in range(3):
    print(M[i])
    lion
    tigre
    monkey
```

The iteration can be over different types of lists and sub-lists:

```
In [50]: for animal in M[1:3]:
```

        print(animal)
    ```
In [51]: for i in [1,2,3]:
    print(i)
    1
```

    tigre
    monkey
    In [52]: | for i in range (4): |
| :---: |
| print(i) |
| $\begin{array}{c}0 \\ 1 \\ 2 \\ 3\end{array}$ |

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| print(i) |
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In [53]: $\left.\begin{aligned} \text { for i in range(3): } \\ \text { print(M[i]) }\end{aligned} \right\rvert\,$
In [56]: for i in range ( len ( M )):
print (" The animal \{\} is a \{\}". format (i , M [ i ]))
The animal 0 is a lion
The animal 1 is a tigre
The animal 2 is a monkey
The animal 3 is a zebra
The animal 4 is a bear
The animal 5 is a snake

## Guess!!

```
>> for x in range(5):
    print(x)
```


## Guess!!

>>> for $x$ in range(5): print(x)<br>0<br>1<br>2<br>3<br>4

## Guess!!

```
>>> for x in range(2,19,3):
    print(x)
```

| $\ggg$ | for $x$ in range $(2,19,3):$ |
| :--- | :--- |
| $\ldots$ | print $(x)$ |
| $\ldots$ |  |
| 2 |  |
| 5 |  |
| 8 |  |
| 11 |  |
| 14 |  |
| 17 |  |

## Guess!!

>>> for $x$ in range $(2,19,3)$ : print (2*x)

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-.

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    print ( animal )
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print (" it is over ")
```

lion
lionlion tigre tigretigre monkey
monkeymonkey
zebra
zebrazebra
bear
bearbear
snake
snakesnake
it is over

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tigre
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    print(i)
```

```
```

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```
```

In [52]: for i in range(4):
print(i)

```
```

    print(i)
    ```
```

    0
    1
2
3

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    print(animal)
```

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    print(i)
1
```

tigre monkey

```
In [52]: for i in range(4):
    print(i)
0
1
2
3
```

```
In [53]: for i in range(3):
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    lion
    tigre
    monkey
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        print(animal)
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    1
```

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    monkey
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| :---: |
| print(i) |
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The animal 0 is a lion
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## Guess!!

Explain the following code and guess the result:

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```
>>> result=[]
>> for i in range(6):
        result.append(i+3)
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```
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3, 4, 5, 6, 7, 8]
```


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Write it in a function

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>> result
3, 4, 5, 6, 7, 8]
```

Write it in a function

```
>>> def guess(n):
    l=[]
    for i in range(n):
            l.append(i)
    return(l)
```


## Guess!!

Explain the following code and guess the result:

```
>>> result=[]
>>> for i in range(6):
    result.append(i+3)
```

```
>> result
3, 4, 5, 6, 7, 8]
```

Write it in a function

```
>>> def guess(n):
    l=[]
    for i in range(n):
            l.append(i)
    return(l)
```

```
>>> guess(5)
[0, 1, 2, 3, 4]
>>> guess(10)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
>>> guess(15)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]
```


## Guess!!

Explain the following code and guess the result:

```
>>> result=[]
>>> for i in range(6):
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```

```
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[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
>>> guess(15)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]
```

What does each of the codes do? What is the diffrence between them?

## Guess!!

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```
>>> result=[]
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    result.append(i+3)
```

```
>> result
3, 4, 5, 6, 7, 8]
```

Write it in a function

```
>>> def guess(n):
    l=[]
    for i in range(n):
        l.append(i)
    return(l)
```

```
>>> guess(5)
[0, 1, 2, 3, 4]
>>> guess(10)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
>>> guess(15)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]
```

What does each of the codes do? What is the diffrence between them?

```
for i in range(6):
    if i >= 2:
        result3.append(i+3)
result4 = [i+3 for i in range(6) if i >= 2]
```


## Exercise 1

Write a function that returns the following sum:

$$
\Sigma_{1}^{n} i=1+2+3 \ldots+n .
$$

## Exercise 1

Write a function that returns the following sum:

$$
\sum_{1}^{n} i=1+2+3 \ldots+n .
$$

```
In [6]: def summation(n):
    a=0
    for i in range(n):
        a+=i+1
    return a
```

In [7]: summation(6)
Out[7]: 21
In [3]: $1+2+3+4+5+6$
Out[3]: 21

## Exercise 2

Try to write the factorial function this time using the Loop FOR:

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Try to write the factorial function this time using the Loop FOR:

```
In [11]: def factorial(n):
    k=1
    for i in range(n):
        k*=(i+1)
    return(k)
In [12]: factorial(5)
Out[12]: 120
In [13]: factorial(7)
Out[13]: 5040
In [14]: factorial(3)
Out[14]: 6
```


## Exercise 3

Write a function that tests the parity of a given number $n$.

## Exercise 3

Write a function that tests the parity of a given number $n$.

```
In [15]: def parity(n):
    if n%2==0:
        return('The number is even')
    else:
        return('The number is odd')
In [16]: parity(5)
Out[16]: 'The number is odd'
In [17]: parity(102)
Out[17]: 'The number is even'
```


## Exercise 4

Create a function that transforms the integer elements of a list into there squares.

## Exercise 4

Create a function that transforms the integer elements of a list into there squares.

```
In [1]: def square(l):
    p=[]
    n=len(l)
    for i in range(n):
        p.append((l[i])**2)
    return p
In [2]: l=[1,2,3,4,5,6,7,8,9,10]
In [3]: square(l)
Out[3]: [1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
```


## Exercise 5

Create a function that displays a list of numerical values of how fast $e^{-x}$ converges to 0 .

## Exercise 5

Create a function that displays a list of numerical values of how fast $e^{-x}$ converges to 0 .

```
In [4]: import math
    def f(n):
        l=[]
        for i in range(n+1):
            l.append(math.e**(-i))
        return l
In [5]: f(5)
Out[5]: [1.0,
    0.36787944117144233,
    0.1353352832366127,
    0.04978706836786395,
    0.018315638888734186,
    0.006737946999085469]
In [6]: f(10)
Out[6]: [1.0,
    0.36787944117144233,
    0.1353352832366127,
    0.04978706836786395,
    0.018315638888734186,
    0.006737946999085469,
    0.0024787521766663594,
    0.0009118819655545166,
    0.00033546262790251196,
    0.00012340980408667962,
    4.5399929762484875e-05]
```


## Loop WHILE

While-loops are loops that repeat while/until a specific condition is met.

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## Loop WHILE

While-loops are loops that repeat while/until a specific condition is met. They can replace for-loops for quicker computation in a lot of cases. Additionally, they do not require a parameter for the amount of repetitions. While-loops simply check the condition statement at each repetition of the loop. If the conditional statement is not met, the loop breaks.

## Loop WHILE

While-loops are loops that repeat while/until a specific condition is met.
They can replace for-loops for quicker computation in a lot of cases. Additionally, they do not require a parameter for the amount of repetitions. While-loops simply check the condition statement at each repetition of the loop. If the conditional statement is not met, the loop breaks. The syntax is: while condition :

```
>>> 1 = []
>>> x = 0
>>> while x < 100:
    1.append(x)
    x += 1
>>> print(1)
[1,2,3,4,5,\ldots 51,52,53,\ldots. 97, 98, 99]
```

```
>>> 1 = []
>>> x = 0
>>> while x < 100:
    1.append(x)
    x += 1
>>> print(l)
[1,2,3,4,5,\ldots 51,52,53, ... 97, 98, 99]
```


## Note

If the fifth line of $x+=1$ hadn't been added, the loop would continue for ever, as the loop would check $0<100$ and then afterwards the loop would append(x) to I.

```
>>> 1 = []
>>> x = 0
>>> while x < 100:
    1. append(x)
    x += 1
>>> print(l)
[1,2,3,4,5,\ldots 51,52,53,\ldots 97, 98, 99]
```


## Note

If the fifth line of $x+=1$ hadn't been added, the loop would continue for ever, as the loop would check $0<100$ and then afterwards the loop would append(x) to I.

Therefore, it is mandatory to think ahead and ensure that the condition that is being checked is continuously changing, otherwise, the loop will never break.

## BREAK

(1) It is a command that can be used in for-loops to imitate a while-loop.

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(1) It is a command that can be used in for-loops to imitate a while-loop.
(2) Break can also be used in a lot of other contexts, as what it does is as soon as a condition is met, it will stop the loop or the function.

```
>>> k = []
>>> for x in range(1000):
    if }x>=100
        break
    else:
        k.append(x)
>>>print(k)
[1,2,3,4,5,\ldots 51,52,53,\ldots. 97, 98, 99]
```


## BREAK

(1) It is a command that can be used in for-loops to imitate a while-loop.
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```
>>> k = []
>>> for x in range(1000):
    if }x>=100
        break
    else:
        k.append(x)
>>>print(k)
[1,2,3,4,5,\ldots 51,52,53,\ldots. 97, 98, 99]
```


## Note

It is seen that both of the two scripts before yield the natural numbers up to, but not including 100. The reason for this is the com- mand break, and hopefully this illustration will help better understand while-loops. While loops are computationally less requiring, and the syntax is easier, thus the for-loop with break should be avoided.

## Exercise 6

Create a script that displays all of the values of $2^{n}<106$, where $n \in N$.

## Exercise 6

Create a script that displays all of the values of $2^{n}<106$, where $n \in N$.

```
def f(x):
    return 2**x
bound = 10**6
n = 0
1 = []
while f(n) < bound:
    l.append(f(n))
    n += 1
print(1)
```

>>>
$[1,2,4,8,16,32,64,128,256,512,1024,2048,4096$,
$8192,16384,32768,65536,131072,262144,524288]$

## Exercise 7

Write a function that deduces the factorial of any number $n \in N$, only using a while loop.

## Exercise 7

Write a function that deduces the factorial of any number $n \in N$, only using a while loop.

```
In [7]: def fac3(n):
    k=1
    while n>=1:
        k=k*n
        n-=1
    return k
```

In [8]: fac3(5)
Out[8]: 120
In [9]: fac3(7)
Out[9]: 5040

## Exercise 8

Write a function using a while loop that gives the $n^{\text {th }}$ element of the following numerical sequence, where $u_{0}=7$ and $u_{n}=2 u_{n-1}+3$ for $n>0$.

## Exercise 8

Write a function using a while loop that gives the $n^{\text {th }}$ element of the following numerical sequence, where $u_{0}=7$ and $u_{n}=2 u_{n-1}+3$ for $n>0$.

```
In [11]: def seq_u(n):
    u=7
    i=0
    while i<n:
        u=2*u+3
        i=i+1
    return u
In [12]: seq_u(0)
Out[12]: 7
In [13]: seq_u(1)
Out[13]: 17
In [14]: seq_u(5)
Out[14]: 317
```


## Exercise 9

Write a function using a while loop that gives the sum of the following elemnts:

$$
S_{n}=\Sigma_{k \geq 0}^{n}\left(\frac{1}{2}\right)^{k} .
$$

## Exercise 9

Write a function using a while loop that gives the sum of the following elemnts:

$$
S_{n}=\sum_{k \geq 0}^{n}\left(\frac{1}{2}\right)^{k} .
$$

```
In [15]: def serie(n):
    s=0.0
    k=0
    while k<=n:
        s=s+((1/2)**k)
        k=k+1
    return s
```

In [16]: serie(3)
Out[16]: 1.875
In [17]: serie(5)
Out[17]: 1.96875
In [18]: serie(1)
Out[18]: 1.5
In [19]: serie(1000)
Out[19]: 2.0

## Exercise 10

Write a function using a while loop that gives the GCD between two positive integers.

## Exercise 10

Write a function using a while loop that gives the GCD between two positive integers.

## Hint

Using Euclide algorithm, we know that to calculate $\operatorname{gcd}(a, b) ; a \geq b$ :

- if $b \neq 0$ then $\operatorname{GCD}(a, b)=G C D(b, a \% b)$
- Else, the $\operatorname{GCD}(a, b)=a$ Remark: we can easily verify that if $a \geq b ; b \neq 0$, then $b \geq a \% b$.


## Exercise 10

Write a function using a while loop that gives the GCD between two positive integers.

| Step n | Dividend | $r_{n-1}$ | Diviseur $r_{n}$ | Equation $r_{n-1}=r_{n} \boldsymbol{q}_{n}+r_{n+1}$ | Quotient $q_{n}$ | Rest $r_{n+1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 21 |  | 15 | $21=15 \times 1+6$ | 1 | 6 |
| 2 | 15 |  | 6 | $15=6 \times 2+3$ | 2 | 3 |
| 3 | 6 | 3 | $6=3 \times 2+0$ | 2 | 0 |  |
| 4 | 3 | 0 | End of the algorithm |  |  |  |

## Exercise 10

Write a function using a while loop that gives the GCD between two positive integers.

$$
\operatorname{gcd}(4052,420)=4
$$

$$
\begin{aligned}
& 4052=9 \times 420+272 \\
& 420=1 \times 272+148 \\
& 272=1 \times 148+124 \\
& 148=1 \times 124+24 \\
& 124=5 \times 24+4 \\
& 24=6 \times 4+0
\end{aligned}
$$

## Solution

```
In [20]: def pgcd(a,b):
    q=a #the quotient
    r=b #divisor
    t=0
    while r!=0:
        t=q%r
        q=r
        r=t
    return q
In [21]: pgcd(9,3)
Out[21]: 3
In [24]: pgcd(15,15)
Out[24]: 15
In [23]: pgcd(56,42)
Out[23]: 14
In [25]: pgcd(4199, 1530)
Out[25]: 17
```


## Exercise 11

Create a scriot that can show the following Triangle:

```
*
**
***
****
*****
******
*******
********
*********
**********
```


## Exercise 11

Create a scriot that can show the following Triangle:

```
In [3]: star='*'
for i in range(10):
    print((i+1)*star)
```

```
*
**
***
****
*****
******
*******
********
*********
**********
```


## Exercise 12

Create a scriot that can show the following Triangle:

```
**********
*********
********
*******
******
*****
****
***
**
*
```


## Exercise 12

Create a scriot that can show the following Triangle:

```
In [3]: star='*'
i=10
while i>=1:
print(i*star)
i-=1
```

```
**********
*********
    ********
    *******
    ******
    *****
    ****
    ***
    **
    *
```


## Exercise 13

Create a scriot that can show the following Triangle:


## Exercise 13

Create a scriot that can show the following Triangle:

```
In [4]: space='
star='*'
for i in range(10):
    print((9-i)*space+(i+1)*star)
```



## Exercise 14

Create a function that can show the following Pyramid for any number of lines:


## Exercise 14

Create a function that can show the following Pyramid for any number of lines:

```
In [11]: def pyra(n):
    space='
    star='*'
    for i in range(1,10):
        print((n-i)*space+(2*i-1)*star)
```



## 2. Matplot library

## Matplotlib.pyplot

- Matplotlib is an enormous module that has functions for plotting data in $2 D$ and $3 D$.


## Matplotlib.pyplot

- Matplotlib is an enormous module that has functions for plotting data in $2 D$ and $3 D$.
- Some of the functions include plot(input, output), ylabel("), title(').


## Matplotlib.pyplot

- Matplotlib is an enormous module that has functions for plotting data in $2 D$ and $3 D$.
- Some of the functions include plot(input, output), ylabel("), title(').
- The syntax and arguments in matplotlib can seem daunting, however, they are in fact very simple with a slight amount of practice.


## Example

(1) import matplotlib.pyplot as plt
plt.plot([1, 2, 3, 4])
plt.ylabel('some numbers')
plt.show()

## Example

(3) import matplotlib.pyplot as plt
plt.plot([1, 2, 3, 4])
plt.ylabel('some numbers')
plt.show()
(2) plt.plot([1, 2, 3, 4], [1, 4, 9, 16])

## Example

(1) import matplotlib.pyplot as plt
plt.plot([1, 2, 3, 4])
plt.ylabel('some numbers')
plt.show()
(O) plt.plot([1, 2, 3, 4], [1, 4, 9, 16])
© plt.plot([1, 2, 3, 4], [1, 4, 9, 16], 'ro')
plt.axis([0, 6, 0, 20])
plt.show()

## Example

(3) import matplotlib.pyplot as plt
plt.plot([1, 2, 3, 4])
plt.ylabel('some numbers')
plt.show()
(3) plt.plot([1, 2, 3, 4], [1, 4, 9, 16])
(3) plt.plot([1, 2, 3, 4], [1, 4, 9, 16], 'ro')
plt.axis([0, 6, 0, 20])
plt.show()
import numpy as $n p$
\# evenly sampled time at 200 ms intervals
$\mathrm{t}=\mathrm{np} . \operatorname{arange}(0 ., 5 ., 0.2)$
\# red dashes, blue squares and green triangles
plt.plot(t, t, 'r--', t, t**2, 'bs', t, t**3, 'g^')
(4) plt. show()

In [2]((-%5Cbigcirc=)): import matplotlib.pyplot as plt ...: $\mathrm{x}=[1,2,3,4,5,6,7]$
$\ldots$. . : $y=[10,8,5,1,5,8,10]$
.... plt.title('Plotting for the first time')
...: plt.ylabel('Output')
....: plt.xlabel('Input')
.... plt.plot(x,y)
[<matplotlib.lines.Line2D at 0x7f96086233d0>]

Plotting for the first time


## More details

The full syntax for this module command is as follows:

$$
\text { plt.plot(input, output, type, label }=\text { label). }
$$

The syntax is trivial, except for 'type'.

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The full syntax for this module command is as follows:

$$
\text { plt.plot(input, output, type, label }=\text { label). }
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The list below shows the possible inputs and the function of the input.

## More details

The full syntax for this module command is as follows:

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\text { plt.plot(input, output, type, label }=\text { label). }
$$

The syntax is trivial, except for 'type'.
The list below shows the possible inputs and the function of the input.
(1) 'r' makes the color red.

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$$
\text { plt.plot(input, output, type, label }=\text { label). }
$$

The syntax is trivial, except for 'type'.
The list below shows the possible inputs and the function of the input.
(1) 'r' makes the color red.
(2) 'b' makes the color blue .

## More details

The full syntax for this module command is as follows:

$$
\text { plt.plot(input, output, type, label }=\text { label). }
$$

The syntax is trivial, except for 'type'.
The list below shows the possible inputs and the function of the input.
(1) 'r' makes the color red.
(2) 'b' makes the color blue .
© ' $k$ ' makes the graph black.

## More details

The full syntax for this module command is as follows:

$$
\text { plt.plot(input, output, type, label }=\text { label). }
$$

The syntax is trivial, except for 'type'.
The list below shows the possible inputs and the function of the input.
(1) 'r' makes the color red.
(2) 'b' makes the color blue.
( ' $k$ ' makes the graph black.
(9) ' $:$ ' makes the line dotted.

## More details

The full syntax for this module command is as follows:

$$
\text { plt.plot(input, output, type, label }=\text { label). }
$$

The syntax is trivial, except for 'type'.
The list below shows the possible inputs and the function of the input.
(1) 'r' makes the color red.
(2) 'b' makes the color blue .
( ' $k$ ' makes the graph black.
(9) ' $:$ ' makes the line dotted.

- 'o' makes the graph a scatter plot with circles.


## More details

The full syntax for this module command is as follows:

$$
\text { plt.plot(input, output, type, label }=\text { label). }
$$

The syntax is trivial, except for 'type'.
The list below shows the possible inputs and the function of the input.
(1) 'r' makes the color red.
(2) 'b' makes the color blue .
( ' $k$ ' makes the graph black.
(9) ' $:$ ' makes the line dotted.

- 'o' makes the graph a scatter plot with circles.
- ' + ' makes the graph a scatter plot with ' + ' as points.


## More details

The full syntax for this module command is as follows:

$$
\text { plt.plot(input, output, type, label }=\text { label). }
$$

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- '+' makes the graph a scatter plot with ' + ' as points.
© 'k:' makes the graph a black dotted line.


## More details

The full syntax for this module command is as follows:

$$
\text { plt.plot(input, output, type, label }=\text { label). }
$$

The syntax is trivial, except for 'type'.
The list below shows the possible inputs and the function of the input.
(1) 'r' makes the color red.
(2) 'b' makes the color blue.
( ' $k$ ' makes the graph black.
(9) ' $:$ ' makes the line dotted.

- 'o' makes the graph a scatter plot with circles.
- '+' makes the graph a scatter plot with ' + ' as points.
( 'k:' makes the graph a black dotted line.
(0) '.r' makes the graph a scatter plot with small red points.


## More details

The full syntax for this module command is as follows:

$$
\text { plt.plot(input, output, type, label }=\text { label). }
$$

The syntax is trivial, except for 'type'.
The list below shows the possible inputs and the function of the input.
(1) 'r' makes the color red.
(2) 'b' makes the color blue.
( ' $k$ ' makes the graph black.
(9) ' $:$ ' makes the line dotted.

- 'o' makes the graph a scatter plot with circles.
- '+' makes the graph a scatter plot with ' + ' as points.
(0) 'k:' makes the graph a black dotted line.
(0) '.r' makes the graph a scatter plot with small red points.

Note that:This list is not a full list of all of the possible commands.

## More details

The full syntax for this module command is as follows:
plt.plot(input, output, type, label = label).

The syntax is trivial, except for 'type'.
The list below shows the possible inputs and the function of the input.
(1) 'r' makes the color red.
(2) 'b' makes the color blue .
( ' $k$ ' makes the graph black.
(9) ' $:$ ' makes the line dotted.

- 'o' makes the graph a scatter plot with circles.
- '+' makes the graph a scatter plot with ' + ' as points.
(0) 'k:' makes the graph a black dotted line.
(0) 'r' makes the graph a scatter plot with small red points.

Note that:This list is not a full list of all of the possible commands.
The arguments inside the string can be concatenated to yield a graph with all of the parameters requested as in 6 and 7 .
[n [7]: import matplotlib.pyplot as plt
import numpy as np
def $f(x)$ :
return $1 / x$
$\mathrm{x}=\mathrm{np}$.arange(0.1, 5, 0.1)
plt.plot (x, $f(x)$, 'b:',label $=$ ' $1 / x$ \$') plt.legend()
[n [7]: import matplotlib.pyplot as plt
import numpy as np
def $f(x)$ :
return $1 / x$
$\mathrm{x}=\mathrm{np}$.arange(0.1, 5, 0.1)
plt.plot(x, $f(x)$, 'b:',label $=$ ' $1 / \mathrm{x}$ \$') plt.legend()


## Some modifications

```
In [15]: import matplotlib.pyplot as plt
    import numpy as np
    def f(x):
    return 1/x
    x= np.arange(0.1, 5, 0.1)
    plt.title('This is a new graph')
    plt.ylabel('Branch of Hyperbola')
    plt.xlabel('X')
    plt.plot(x, f(x), 'r+',label = '1/x')
    plt.legend()
```


## Some modifications

This is a new graph


## Exercise I

Write a script that will produce a red curve of $\sin x$ in the interval of $[-\pi, \pi]$. Ensure that there is a black $x$-axis.

## Exercise I

Write a script that will produce a red curve of $\sin x$ in the interval of $[-\pi, \pi]$. Ensure that there is a black $x$-axis.

In [16]: import numpy as np
import matplotlib. pyplot as plt
def $f(x)$ :
return np. $\sin (x)$
$x=n p . a r a n g e(-n p . p i, n p . p i, 0.01)$
plt.ylabel("y")
plt.xlabel("x")
plt.plot( $x, f(x), ~ " r: ", ~ l a b e l=" s i n ~ x ")$
plt.plot( $x, x^{*} \theta, " k "$, label $\left.=" y=0 "\right)$
plt.legend()

Out[16]: <matplotlib.legend.Legend at 0x7f8651a862e0>


## Exercise II

Make a script that approximates the limit of f as $x \rightarrow 0$ of $\sin (x)$ defined as:

$$
f:] 0,1] \rightarrow \mathbb{R}: x \mapsto \frac{\sin x}{x}
$$

## Exercise II

Make a script that approximates the limit of f as $x \rightarrow 0$ of $\sin (x)$ defined as:

$$
f:] 0,1] \rightarrow \mathbb{R}: x \mapsto \frac{\sin x}{x}
$$

In [32]: import numpy as np

```
import matplotlib.pyplot as plt
```

$\mathrm{N}=100000$
$\mathrm{i}=\mathrm{np}$. arange $(1, \mathrm{~N})$
$\mathrm{x}=1$ /i
$y=n p . \sin (x) / x$
plt.plot(x,y, 'r:', label $=$ "lim of $\sin x / x$ tends to 0.9999999999833329 ")
plt.legend()

Out[32]: <matplotlib.legend.Legend at 0x7f86519491f0>


## Exercise III

Write the code that gives you the following figure.


## Exercise III

Write the code that gives you the following figure.

```
import numpy as np
import matplotlib.pyplot as plt
# Compute the x and y coordinates for points on sine and cosine curves
x = np.arange(0, 3 * np.pi, 0.1)
y_sin = np.sin(x)
y_cos = np.cos(x)
# Plot the points using matplotlib
plt.plot(x, y_sin)
plt.plot(x, y_cos)
plt.xlabel('x axis label')
plt.ylabel('y axis label')
plt.title('Sine and Cosine')
plt.legend(['Sine', 'Cosine'])
plt.show()
```


## Interlude

Create an array of equally spaced 10 data in range 0 to $2 \pi$.

## Interlude

## Create an array of equally spaced 10 data in range 0 to $2 \pi$.

```
In [4]: import numpy as np
l=[]
for i in range(10):
    l.append(i/10*np.pi*2)
print(l)
```

[0.0, $0.6283185307179586,1.2566370614359172,1.8849555921538759,2.5132741228718345,3.141592653589793,3.76991118$ 43077517, 4.39822971502571, 5.026548245743669, 5.654866776461628]

## Interlude

Create an array of equally spaced 10 data in range 0 to $2 \pi$.

```
In [4]: import numpy as np
    l=[]
    for i in range(10):
    l.append(i/10*np.pi*2)
print(l)
[0.0, 0.6283185307179586, 1.2566370614359172, 1.8849555921538759, 2.5132741228718345, 3.141592653589793, 3.76991118
43077517, 4.39822971502571, 5.026548245743669, 5.654866776461628]
```

Another solution is using numpy linspace

```
In [7]: print(np.linspace(0,2*np.pi,11))
[0. 0.62831853 1.25663706 1.88495559 2.51327412 3.14159265
    3.76991118 4.39822972 5.02654825 5.65486678 6.28318531]
```


## Exercise IV

This example uses following formula to generate heart shape $x$ and $y$ coordinates:

$$
x=16 * \sin ^{3}(\theta) \text { and } y=13 * \cos (\theta)-5 * \cos (2 \theta)-2 * \cos (3 \theta)-\cos (4 \theta)
$$

## Exercise IV

This example uses following formula to generate heart shape $x$ and $y$ coordinates:

$$
x=16 * \sin ^{3}(\theta) \text { and } y=13 * \cos (\theta)-5 * \cos (2 \theta)-2 * \cos (3 \theta)-\cos (4 \theta)
$$

\# Python program to Plot Perfect Heart Shape
\# importing libraries
import numpy as np
from matplotlib import pyplot as plt
\# Creating equally spaced 100 data in range 0 to $2^{*} p i$
theta $=n p . \operatorname{linspace}(0,2$ * np.pi, 100)
\# Generating $x$ and $y$ data
$x=16{ }^{*}(n p \cdot \sin ($ theta $) * * 3)$
$y=13$ * np. cos (theta) - $5^{*} n p \cdot \cos \left(2^{*}\right.$ theta) - 2 * np. $\cos \left(3^{*}\right.$ theta) - np. cos ( $4^{*}$ theta)
\# Plotting
plt.plot (x, y)
plt.title('Heart Shape')
plt.show( )

## Exercise IV

This example uses following formula to generate heart shape $x$ and $y$ coordinates:

$$
x=16 * \sin ^{3}(\theta) \text { and } y=13 * \cos (\theta)-5 * \cos (2 \theta)-2 * \cos (3 \theta)-\cos (4 \theta)
$$



This part is for the documentation. Check this section in particular: matplotlib.pyplot.plot(*args, **kwargs)

## Subplots

You can plot different things in the same figure using the subplot function. Here is an example:

## Subplots

You can plot different things in the same figure using the subplot function. Here is an example:

```
import numpy as np
import matplotlib.pyplot as plt
# Compute the x and y coordinates for points on sine and cosine curves
x = np.arange(0, 3 * np.pi, 0.1)
y_sin = np.sin(x)
y_cos=np.cos(x)
# Set up a subplot grid that has height }2\mathrm{ and width 1,
# and set the first such subplot as active.
plt.subplot(2, 1, 1)
# Make the first plot
plt.plot(x, y_sin)
plt.title('Sine')
# Set the second subplot as active, and make the second plot.
plt.subplot(2, 1, 2)
plt.plot(x, y_cos)
plt.title('Cosine')
# Show the figure.
plt.show()
```


## Subplots

You can plot different things in the same figure using the subplot function. Here is an example:


## Second way with subplots

```
import matplotlib.pyplot as plt
import numpy as np
# Some example data to display
x = np.linspace(0, 2 * np.pi, 400)
y = np.sin(x ** 2)
```

subplots() without arguments returns a Figure and a single Axes.

```
fig, ax = plt.subplots()
ax.plot(x, y)
ax.set_title('A single plot')
```



The first two optional arguments of pyplot.subplots define the number of rows and columns of the subplot grid.

The first two optional arguments of pyplot.subplots define the number of rows and columns of the subplot grid.

```
fig, axs = plt.subplots(2)
fig.suptitle('Vertically stacked subplots')
axs[0].plot(x, y)
axs[1].plot(x, -y)
```

Vertically stacked subplots



To obtain side-by-side subplots, pass parameters 1,2 for one row and two columns.

To obtain side-by-side subplots, pass parameters 1,2 for one row and two columns.

```
fig, (ax1, ax2) = plt.subplots(1, 2)
fig.suptitle('Horizontally stacked subplots')
ax1.plot(x, y)
ax2.plot(x, -y)
```

Horizontally stacked subplots


## Stacking subplots in two directions

```
fig, axs = plt.subplots(2, 2)
axs[0, 0].plot(x, y)
axs[0, 0].set_title('Axis [0,0]')
axs[0, 1].plot(x, y, 'tab:orange')
axs[0, 1].set_title('Axis [0,1]')
axs[1, 0].plot(x, -y, 'tab:green')
axs[1, 0].set_title('Axis [1,0]')
axs[1, 1].plot(x, -y, 'tab:red')
axs[1, 1].set_title('Axis [1,1]')
for ax in axs.flat:
    ax.set(xlabel='x-label', ylabel='y-label')
# Hide x labels and tick labels for top plots and y ticks for right plots.
for ax in axs.flat:
    ax.label_outer()
```

Stacking subplots in two directions


By default, each Axes is scaled individually. Thus, if the ranges are different the tick values of the subplots do not align.

By default, each Axes is scaled individually. Thus, if the ranges are different the tick values of the subplots do not align.

```
fig, (ax1, ax2) = plt.subplots(2)
fig.suptitle('Axes values are scaled individually by default')
ax1.plot(x, y)
ax2.plot(x+1, -y)
```

By default, each Axes is scaled individually. Thus, if the ranges are different the tick values of the subplots do not align.

```
fig, (ax1, ax2) = plt.subplots(2)
fig.suptitle('Axes values are scaled individually by default')
ax1.plot(x, y)
ax2.plot(x + 1, -y)
```

Axes values are scaled individually by default



You can use sharex or sharey to align the horizontal or vertical axis.

You can use sharex or sharey to align the horizontal or vertical axis.

```
fig, (ax1, ax2) = plt.subplots(2, sharex=True)
fig.suptitle('Aligning x-axis using sharex')
ax1.plot(x, y)
ax2.plot(x + 1, -y)
```

You can use sharex or sharey to align the horizontal or vertical axis.

```
fig, (ax1, ax2) = plt.subplots(2, sharex=True)
fig.suptitle('Aligning x-axis using sharex')
ax1.plot(x, y)
ax2.plot(x + 1, -y)
```

Aligning $x$-axis using sharex



You can use sharex or sharey to align the horizontal or vertical axis.

```
fig, axs = plt.subplots(3, sharex=True, sharey=True)
fig.suptitle('Sharing both axes')
axs[0].plot(x, y ** 2)
axs[1].plot(x, 0.3 * y, 'o')
axs[2].plot(x, y, '+')
```

Sharing both axes


You can use sharex or sharey to align the horizontal or vertical axis.

```
fig, axs = plt.subplots(3, sharex=True, sharey=True)
fig.suptitle('Sharing both axes')
axs[0].plot(x, y ** 2)
axs[1].plot(x, 0.3 * y, 'o')
axs[2].plot(x, y, '+')
```

Sharing both axes


The parameter gridspec_kw of pyplot.subplots controls the grid properties. For example, we can reduce the height between vertical subplots using gridspec_kw=\{'hspace': 0\}. label_outer is a handy method to remove labels and ticks from subplots that are not at the edge of the grid.

The parameter gridspec _kw of pyplot.subplots controls the grid properties. For example, we can reduce the height between vertical subplots using gridspec_kw=\{'hspace': 0\}. label_outer is a handy method to remove labels and ticks from subplots that are not at the edge of the grid.

```
fig, axs = plt.subplots(3, sharex=True, sharey=True, gridspec_kw={'hspace': 0})
fig.suptitle('Sharing both axes')
axs[0].plot(x, y ** 2)
axs[1].plot(x, 0.3 * y, 'o')
axs[2].plot(x, y, '+')
# Hide x labels and tick labels for all but bottom plot.
for ax in axs:
    ax.label_outer()
```

The parameter gridspec kw of pyplot.subplots controls the grid properties. For example, we can reduce the height between vertical subplots using gridspec_kw=\{'hspace': 0\}.
label_outer is a handy method to remove labels and ticks from subplots that are not at the edge of the grid.

Sharing both axes


Apart from True and False, both sharex and sharey accept the values 'row' and 'col' to share the values only per row or column.

Apart from True and False, both sharex and sharey accept the values 'row' and 'col' to share the values only per row or column.

```
fig, axs = plt.subplots(2, 2, sharex='col', sharey='row',
    gridspec_kw={'hspace': 0, 'wspace': 0})
(a\times1, ax2), (a\times3, a\times4) = axs
fig.suptitle('Sharing x per column, y per row')
ax1.plot(x, y)
ax2.plot(x, y**2, 'tab:orange')
ax3.plot(x + 1, -y, 'tab:green')
ax4.plot(x + 2, -y**2, 'tab:red')
for ax in axs.flat:
    ax.label_outer()
```

Apart from True and False, both sharex and sharey accept the values 'row' and 'col' to share the values only per row or column.

Sharing $\times$ per column, y per row


### 3.3D plotting

## 3D plotting

Importing the mplot3d library enables 3D plotting.
You can create 3D axes by passing projection="3d" to any of the regular axes creation functions.

## 3D plotting

Importing the mplot3d library enables 3D plotting.
You can create 3D axes by passing projection="3d" to any of the regular axes creation functions.

```
from mpl_toolkits import mplot3d
import numpy as np
import matplotlib.pyplot as plt
fig = plt.figure()
ax = plt.axes(projection="3d")
plt.show()
```


## 3D plotting

Importing the mplot3d library enables 3D plotting. You can create 3D axes by passing projection="3d" to any of the regular axes creation functions.


## Plotting a 3d curve

(2)
(3)

$$
\begin{align*}
& x=\cos (z)  \tag{1}\\
& y=\sin (z) \\
& z=z
\end{align*}
$$

## Plotting a 3d curve

(1)
(2)
(3)
$x=\cos (z)$
$y=\sin (z)$
$z=z$

```
fig = plt.figure()
ax = plt.axes(projection="3d")
z_line = np.linspace(0, 15, 1000)
x_line = np.cos(z_line)
y_line = np.sin(z_line)
ax.plot3D(x_line, y_line, z_line, 'gray')
plt.show()
```


## Plotting a 3d curve

(1)
(2)
(3)

$$
\begin{aligned}
& x=\cos (z) \\
& y=\sin (z) \\
& z=z
\end{aligned}
$$



## Example: Plotting a 3d curve

Draw the 3D plot of
(4)
(5)
(6)

$$
\begin{aligned}
& x=z \times \sin (20 z) \\
& y=z \times \cos (20 z) \\
& z=z
\end{aligned}
$$

## Example: Plotting a 3d curve

Draw the 3D plot of
(6)

```
    x = z < \operatorname{sin}(20z)
    y = z\times\operatorname{cos(20z)}
    z = z
fig = plt.figure()
ax = plt.axes(projection='3d')
z = np.linspace(0, 1, 100)
x = z * np.sin(20 * z)
y = z * np.cos(20 * z)
ax.plot3D(x, y, z, 'red')
```


## Example: Plotting a 3d curve

Draw the 3D plot of
(4)

$$
\begin{aligned}
x & =z \times \sin (20 z) \\
y & =z \times \cos (20 z) \\
z & =z
\end{aligned}
$$

(5)
(6)


## Plotting a 3d curve and some points

```
fig = plt.figure()
ax = plt.axes(projection="3d")
z_line = np.linspace(0, 15, 1000)
x_line = np.cos(z_line)
y_line = np.sin(z_line)
ax.plot3D(x_line, y_line, z_line, 'gray')
z_points = 15 * np.random.random(100)
x_points = np.cos(z_points) + 0.1 * np.random.randn(100)
y_points = np.sin(z_points) + 0.1 * np.random.randn(100)
ax.scatter3D(x_points, y_points, z_points, c=z_points, cmap='hsv');
plt.show()
```


## Plotting a 3d curve and some points



## 1.Surfaces

## Plotting a surface (1)

This is a 3 step process.
First step is to define the function and to generate enough points to estimate the surface.

```
def z_function(x, y):
        return np.sin(np.sqrt(x ** 2 + y ** 2))
x = np.linspace(-6, 6, 30)
y = np.linspace(-6, 6, 30)
X, Y = np.meshgrid(x, y)
8 Z = z_function(X, Y)
```


## Plotting a surface (2)

The second step is to plot a wire-frame - this is our estimate of the surface.

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```
1 fig = plt.figure()
2 ax = plt.axes(projection="3d")
3 ax.plot_wireframe(X, Y, Z, color='green')
4 ax.set_xlabel('x')
5 ax.set_ylabel('y')
6 ax.set_zlabel('z')
7
8 plt.show()
```


## Plotting a surface (2)

The second step is to plot a wire-frame - this is our estimate of the surface.


## Plotting a surface (3)

Finally, we'll project our surface onto our wire-frame estimate and extrapolate all of the points.

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Finally, we'll project our surface onto our wire-frame estimate and extrapolate all of the points.

1 ax = plt.axes(projection='3d')
2 ax.plot_surface(X, Y, Z, cmap='winter')
3 ax.set_title('surface');

## Plotting a surface (3)

Finally, we'll project our surface onto our wire-frame estimate and extrapolate all of the points.


Colormaps: https://matplotlib.org/3.1.1/tutorials/colors/colormaps.html

Draw the following surfaces:

$$
\begin{gathered}
\mathrm{f}: \mathbb{R}^{2} \rightarrow \mathbb{R}:(x, y) \rightarrow x^{2}+y^{2} \\
\mathrm{~g}: \mathbb{R}^{2} \rightarrow \mathbb{R}:(x, y) \rightarrow 4 x-2 y+3
\end{gathered}
$$

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g: \mathbb{R}^{2} \rightarrow \mathbb{R}:(x, y) \rightarrow 4 x-2 y+3
\end{gathered}
$$

In [19]: \%matplotlib notebook
import matplotlib.pyplot as plt
import numpy as np
$a x=p l t . a x e s\left(p r o j e c t i o n={ }^{\prime} \mathbf{3 d}^{\prime}\right)$
$x=n p . a r a n g e(-4,4,0.01)$
$y=n p$. arange $(-4,4,0.01)$
$X, Y=n p . m e s h g r i d(x, y)$
def $f(x, y)$ :
return $x^{* * 2}+y^{* * 2}$
def $g(x, y)$ :
return 4*x - 2*y + 3
$Z 1=f(X, Y)$
$Z 2=g(X, Y)$
ax.plot_surface(X, Y, Z1, color ='r')
ax.plot_surface(X, Y, Z2, color ='b')

Draw the following surfaces:

$$
\begin{gathered}
f: \mathbb{R}^{2} \rightarrow \mathbb{R}:(x, y) \rightarrow x^{2}+y^{2} \\
g: \mathbb{R}^{2} \rightarrow \mathbb{R}:(x, y) \rightarrow 4 x-2 y+3
\end{gathered}
$$

>>>


Draw the following surface:

$$
\begin{gathered}
x=\sqrt{30} \cos (t)+3 \\
y=\sqrt{30} \sin (t)-3 \\
z=6 x-6 y+12
\end{gathered}
$$

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z=6 x-6 y+12
\end{gathered}
$$

```
import numpy as np
import matplotlib.pyplot as plt
ax = plt.axes(projection = '3d')
\(\mathrm{t}=\mathrm{np} . \operatorname{arange}(-\mathrm{np} . \mathrm{pi}, \mathrm{np} . \mathrm{pi}, 0.01)\)
x_line \(=\) np.sqrt(30) * np.cos(t) +3
y_line \(=\) np.sqrt(30) \(*\) np.sin(t) -3
z_line \(=6 * x\) line \(-6 * y\) _line +12
ax.plot3D(x_line,y_line, z_line, 'r')
```

Draw the following surface:

$$
\begin{gathered}
x=\sqrt{30} \cos (t)+3 \\
y=\sqrt{30} \sin (t)-3 \\
z=6 x-6 y+12
\end{gathered}
$$

>>>


## Example

plot the surface of the function $f(x, y)$ in the domain of $D(f)=[10,10]^{2} \subset \mathbb{R}$ from a good view. Where $f$ is, $f: D(f)^{2} \rightarrow \mathbb{R}:(x, y) \rightarrow e^{x}\left(x^{2}-y^{3}\right)$

## Example

plot the surface of the function $f(x, y)$ in the domain of $D(f)=[10,10]^{2} \subset \mathbb{R}$ from a good view. Where $f$ is, $f: D(f)^{2} \rightarrow \mathbb{R}:(x, y) \rightarrow e^{x}\left(x^{2}-y^{3}\right)$

In [26]: import numpy as np import matplotlib.pyplot as plt def $f(x, y)$ :
return np.e**x * (x**2 - $\mathrm{y}^{* * 3}$ )
$x=n p . a r a n g e(-10,10,0.1)$
$y=n p . a r a n g e(-10,10,0.1)$
$\mathrm{X}, \mathrm{Y}=\mathrm{np}$. meshgrid $(\mathrm{x}, \mathrm{y})$
$Z=f(X, Y)$
ax = plt.axes(projection = '3d')
ax.plot_surface(X, Y, Z, alpha $=0.35$, color $=$ ' $r^{\prime}$ )
ax.view_init(30,140)

## Example

plot the surface of the function $f(x, y)$ in the domain of $D(f)=[10,10]^{2} \subset \mathbb{R}$ from a good view. Where $f$ is, $f: D(f)^{2} \rightarrow \mathbb{R}:(x, y) \rightarrow e^{x}\left(x^{2}-y^{3}\right)$
>>>


## 1.Bissection Theorem

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The Bisection Method is a means of numerically approximating a solution to an equation: $f(x)=0$

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The Bisection Method is a means of numerically approximating a solution to an equation: $f(x)=0$
The fundamental mathematical principle underlying the Bisection Method is the Intermediate Value Theorem.

## Theorem

Let $f:[a, b] \rightarrow[a, b]$ be a continuous function. Suppose that $d$ is any value between $f(a)$ and $f(b)$. Then there is a $c ; a<c<b$, such that $f(c)=d$.

The Intermediate Value Theorem implies that if $f(a) f(b)<0$, then there is a point c; $a<c<b$ such that $f(c)=0$.

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Let $f:[a, b] \rightarrow[a, b]$ be a continuous function. Suppose that $d$ is any value between $f(a)$ and $f(b)$. Then there is a $c ; a<c<b$, such that $f(c)=d$.

The Intermediate Value Theorem implies that if $f(a) f(b)<0$, then there is a point c; $a<c<b$ such that $f(c)=0$.

Thus if we have a continuous function $f$ on an interval $[a, b]$ such that $f(a) f(b)<0$, then $f(x)=0$ has a solution in that interval.

## Algorithm

The Intermediate Value Theorem not only guarantees a solution to the equation, but it also provides a means of numerically approximating a solution to arbitrary accuracy.

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Proceed as follows:
(1) Let $\epsilon>0$ be the upper bound for the error required of the answer

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Proceed as follows:
(1) Let $\epsilon>0$ be the upper bound for the error required of the answer
(2) Compute $c=\frac{a+b}{2}$ and $d=f(c) \times f(a)$.

## Algorithm

The Intermediate Value Theorem not only guarantees a solution to the equation, but it also provides a means of numerically approximating a solution to arbitrary accuracy.
Proceed as follows:
(3) Let $\epsilon>0$ be the upper bound for the error required of the answer
(c) Compute $c=\frac{a+b}{2}$ and $d=f(c) \times f(a)$.
(3) If $d<0$, then let $b=c$ and $a=a$.

## Algorithm

The Intermediate Value Theorem not only guarantees a solution to the equation, but it also provides a means of numerically approximating a solution to arbitrary accuracy.
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(1) Let $\epsilon>0$ be the upper bound for the error required of the answer
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(0) If $d>0$, then let $a=c$ and $b=b$.

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(0) If $d=0$, then $c$ is a solution of $f(x)=0$ and a solution has been found to the required accuracy.

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- The new interval $[a, b]$ will then be half the length of the original $[a, b]$ and will contain a point $x \in[a, b]$ such that $f(x)=0$.


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Proceed as follows:
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(0) If $d>0$, then let $a=c$ and $b=b$.
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- The new interval $[a, b]$ will then be half the length of the original $[a, b]$ and will contain a point $x \in[a, b]$ such that $f(x)=0$.

Repeat 2) until either an exact solution is found in 5) or until at step 4) half the length of $[a, b]$ is less than, $\frac{b-a}{2}<2$.

## Video+Example1

Find the approximate intersection between $h(x)=x^{3}$ and $g(x)=x+1$.

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## Solution

Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$.

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Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

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Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

$$
\begin{aligned}
& 1^{\text {st }} \text { iteration : } \\
& \text { Here } f(1)=-1<0 \text { and } f(2)=5>0 \\
& \therefore \text { Now, Root lies between } 1 \text { and } 2 \\
& x_{0}=\frac{1+2}{2}=1.5 \\
& f\left(x_{0}\right)=f(1.5)=0.875>0
\end{aligned}
$$

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Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$. So let us find a root of an equation $f(x)=x^{3}-x-1$ using Bisection method.
Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

$$
\left\lvert\, \begin{aligned}
& 2^{\text {nd }} \text { iteration : } \\
& \text { Here } f(1)=-1<0 \text { and } f(1.5)=0.875>0 \\
& \therefore \text { Now, Root lies between } 1 \text { and } 1.5 \\
& x_{1}=\frac{1+1.5}{2}=1.25 \\
& f\left(x_{1}\right)=f(1.25)=-0.29688<0
\end{aligned}\right.
$$

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Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

$$
\begin{aligned}
& 3^{\text {rd }} \text { iteration : } \\
& \text { Here } f(1.25)=-0.29688<0 \text { and } f(1.5)=0.875>0 \\
& \therefore \text { Now, Root lies between } 1.25 \text { and } 1.5 \\
& x_{2}=\frac{1.25+1.5}{2}=1.375 \\
& f\left(x_{2}\right)=f(1.375)=0.22461>0
\end{aligned}
$$

## Video+Example1

Find the approximate intersection between $h(x)=x^{3}$ and $g(x)=x+1$.

## Solution

Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$. So let us find a root of an equation $f(x)=x^{3}-x-1$ using Bisection method.
Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

$4^{\text {th }}$ iteration :
Here $f(1.25)=-0.29688<0$ and $f(1.375)=0.22461>0$
$\therefore$ Now, Root lies between 1.25 and 1.375

$$
\begin{aligned}
& x_{3}=\frac{1.25+1.375}{2}=1.3125 \\
& f\left(x_{3}\right)=f(1.3125)=-0.05151<0
\end{aligned}
$$

## Video+Example1

Find the approximate intersection between $h(x)=x^{3}$ and $g(x)=x+1$.

## Solution

Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$. So let us find a root of an equation $f(x)=x^{3}-x-1$ using Bisection method.
Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

$5^{\text {th }}$ iteration :
Here $f(1.3125)=-0.05151<0$ and $f(1.375)=0.22461>0$
$\therefore$ Now, Root lies between 1.3125 and 1.375

$$
\begin{aligned}
& x_{4}=\frac{1.3125+1.375}{2}=1.34375 \\
& f\left(x_{4}\right)=f(1.34375)=0.08261>0
\end{aligned}
$$

## Video+Example1

Find the approximate intersection between $h(x)=x^{3}$ and $g(x)=x+1$.

## Solution

Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$. So let us find a root of an equation $f(x)=x^{3}-x-1$ using Bisection method.
Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

$$
\begin{aligned}
& 6^{\text {th }} \text { iteration : } \\
& \text { Here } f(1.3125)=-0.05151<0 \text { and } f(1.34375)= \\
& \therefore \text { Now, Root lies between } 1.3125 \text { and } 1.34375 \\
& x_{5}=\frac{1.3125+1.34375}{2}=1.32812 \\
& f\left(x_{5}\right)=f(1.32812)=0.01458>0
\end{aligned}
$$

$$
\text { Here } f(1.3125)=-0.05151<0 \text { and } f(1.34375)=0.08261>0
$$

## Video+Example1

Find the approximate intersection between $h(x)=x^{3}$ and $g(x)=x+1$.

## Solution

Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$. So let us find a root of an equation $f(x)=x^{3}-x-1$ using Bisection method.
Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

$$
\begin{aligned}
& 7^{\text {th }} \text { iteration: } \\
& \text { Here } f(1.3125)=-0.05151<0 \text { and } f(1.32812)=0.01458>0 \\
& \therefore \text { Now, Root lies between } 1.3125 \text { and } 1.32812 \\
& x_{6}=\frac{1.3125+1.32812}{2}=1.32031 \\
& f\left(x_{6}\right)=f(1.32031)=-0.01871<0
\end{aligned}
$$

## Video+Example1

Find the approximate intersection between $h(x)=x^{3}$ and $g(x)=x+1$.

## Solution

Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$. So let us find a root of an equation $f(x)=x^{3}-x-1$ using Bisection method.
Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

$$
\begin{aligned}
& 8^{\text {th }} \text { iteration : } \\
& \text { Here } f(1.32031)=-0.01871<0 \text { and } f(1.32812)=0.01458>0 \\
& \therefore \text { Now, Root lies between } 1.32031 \text { and } 1.32812 \\
& x_{7}=\frac{1.32031+1.32812}{2}=1.32422 \\
& f\left(x_{7}\right)=f(1.32422)=-0.00213<0
\end{aligned}
$$

## Video+Example1

Find the approximate intersection between $h(x)=x^{3}$ and $g(x)=x+1$.

## Solution

Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$. So let us find a root of an equation $f(x)=x^{3}-x-1$ using Bisection method.
Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

```
9th iteration :
Here f(1.32422)=-0.00213<0 and f(1.32812) = 0.01458>0
\therefore Now, Root lies between 1.32422 and 1.32812
x}=\frac{1.32422+1.32812}{2}=1.3261
f(\mp@subsup{x}{8}{})=f(1.32617)=0.00621>0
```


## Video+Example1

Find the approximate intersection between $h(x)=x^{3}$ and $g(x)=x+1$.

## Solution

Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$. So let us find a root of an equation $f(x)=x^{3}-x-1$ using Bisection method.
Here

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f(x)$ | -1 | -1 | 5 |

$$
\begin{aligned}
& 10^{\text {th }} \text { iteration : } \\
& \text { Here } f(1.32422)=-0.00213<0 \text { and } f(1.32617)=0.00621>0 \\
& \therefore \text { Now, Root lies between } 1.32422 \text { and } 1.32617 \\
& x_{9}=\frac{1.32422+1.32617}{2}=1.3252 \\
& f\left(x_{9}\right)=f(1.3252)=0.00204>0
\end{aligned}
$$

## Video+Example1

Find the approximate intersection between $h(x)=x^{3}$ and $g(x)=x+1$.

## Solution

Find the intersection between $h(x)$ and $g(x)$ means to find an $x_{0}$ such that $h\left(x_{0}\right)=g\left(x_{0}\right)$ i.e $h\left(x_{0}\right)-g\left(x_{0}\right)=0$. This means that $x_{0}$ is a root of $h(x)-g(x)$. So let us find a root of an equation $f(x)=x^{3}-x-1$ using Bisection method.
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& 11^{\text {th }} \text { iteration : } \\
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& \therefore \text { Now, Root lies between } 1.32422 \text { and } 1.3252 \\
& x_{10}=\frac{1.32422+1.3252}{2}=1.32471 \\
& f\left(x_{10}\right)=f(1.32471)=-0.00005<0
\end{aligned}
$$

## Approximate root of the equation $x^{3}-x-1=0$ using Bisection mehtod is 1.32471

| $n$ | $a$ | $f(a)$ | $b$ | $f(b)$ | $c=\frac{a+b}{2}$ | $f(c)$ | Update |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | -1 | 2 | 5 | 1.5 | 0.875 | $b=c$ |
| 2 | 1 | -1 | 1.5 | 0.875 | 1.25 | -0.29688 | $a=c$ |
| 3 | 1.25 | -0.29688 | 1.5 | 0.875 | 1.375 | 0.22461 | $b=c$ |
| 4 | 1.25 | -0.29688 | 1.375 | 0.22461 | 1.3125 | -0.05151 | $a=c$ |
| 5 | 1.3125 | -0.05151 | 1.375 | 0.22461 | 1.34375 | 0.08261 | $b=c$ |
| 6 | 1.3125 | -0.05151 | 1.34375 | 0.08261 | 1.32812 | 0.01458 | $b=c$ |
| 7 | 1.3125 | -0.05151 | 1.32812 | 0.01458 | 1.32031 | -0.01871 | $a=c$ |
| 8 | 1.32031 | -0.01871 | 1.32812 | 0.01458 | 1.32422 | -0.00213 | $a=c$ |
| 9 | 1.32422 | -0.00213 | 1.32812 | 0.01458 | 1.32617 | 0.00621 | $b=c$ |
| 10 | 1.32422 | -0.00213 | 1.32617 | 0.00621 | 1.3252 | 0.00204 | $b=c$ |
| 11 | 1.32422 | -0.00213 | 1.3252 | 0.00204 | 1.32471 | -0.00005 | $a=c$ |

## Example 2

write a script that finds the approximate intersection between $f$ and $g$, through use of the bisection theorem. Additionally, graph $f(x)$ and $g(x)$ and their intersection.

$$
f: \mathbb{R} \rightarrow \mathbb{R}: x \rightarrow \cos ^{2}(x), g: \mathbb{R} \rightarrow \mathbb{R}: x \rightarrow x^{2}
$$

## Solution

(1) Start by defining the function $i(x)$, whose root is going to be the value of intersection required.
(2) Define the function that is going to use the bissection method and has 4 inputs: $i, a, b, \epsilon$, where a and b are the boundaries of your interval of study and $\epsilon$ is the smallest value of accepted error.
(3) With the IF test, check if we have a root in this interval or not.
(1) If not, keep repeating the bissection method as long as $|a-b| / 2>\epsilon$
( Stop the test when the image of the midpoint is equal to 0 .

- Draw the two functions with their intersection point.


## Solution

```
import numpy as np
def i(x):
    return (np.cos(x))**2 - x**2
```


## Solution

def bisection method $(f, a, b$, epsilon):
if $f(a) * f(b)>0:$
return 'no root exists'

## Solution

```
def bisection method(f,a,b,epsilon):
    if f(a)*f(b)>0:
                return 'no root exists'
    else:
        while np.absolute(a-b)/2 > epsilon:
                        midpoint = (a+b)/2
```


## Solution

```
def bisection_method(f,a,b,epsilon):
    if f(a)*f(b) > 0:
        return 'no root exists'
    else:
        while np.absolute(a-b)/2 > epsilon:
            midpoint = (a+b)/2
            if f(midpoint) == 0:
                        return midpoint
```


## Solution

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            if f(midpoint) == 0:
                    return midpoint
                elif f(midpoint)*f(a)<0:
                        b= midpoint
```


## Solution

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                            return midpoint
                elif f(midpoint)*f(a) < 0:
                        b= midpoint
                else:
                    a = midpoint
            return midpoint
```


## Solution

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import numpy as np
def i(x):
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def bisectionmethod(f,a,b,epsilon):
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        while np.absolute(a-b)/2 > epsilon:
            midpoint = (a+b)/2
            if f(midpoint) == 0:
                    return midpoint
            elif f(midpoint)*f(a) < 0:
                b= midpoint
            else:
                        a = midpoint
            return midpoint
def f(x):
            return (np.cos(x))**2
def g(x):
    return x**2
x = np.arange (0,2,0.01)
intercept = bisection_method(i,0,2,10**-6)
plt.plot(x, f(x), 'b', label = '$f(x)$')
plt.plot(x, g(x), 'g', label = '$g(x)$')
plt.plot(intercept, f(intercept), 'or', label =
str(intercept))
plt.legend()
```


## Solution



## 1. Widgets on Jupiter

## What are widgets?

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Widgets are eventful python objects that have a representation in the browser, often as a control like a slider, textbox, etc.

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## What can they be used for?

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## What can they be used for?

You can use widgets to build interactive GUlsfor your notebooks. You can also use widgets to synchronizestateful and stateless information between Python and JavaScript.

## Using widgets

Baby steps.

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```
[2]: widgets.IntSlider()
```



- You can also explicitly display the widget using display(...).
[3]: from IPython.display import display $w=$ widgets.IntSlider() display(w)
(0) If you display the same widget twice, the displayed instances in the front-end will remain in sync with each other. Try dragging the slider below and watch the slider above.
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- You can close a widget by calling its close() method.
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(1) In addition to value, most widgets sharekeys, description, and disabled. To see the entire list of synchronized, stateful properties of any specific widget, you can query the keys property.

```
[10]: w.keys
[10]: [',_dom_classes',
    '_model_module',
    '_model_module_version',
    '_model_name',
    ',_view_count',
    '_view_module',
    '__view_module_version',
    '_view_name',
    'continuous_update',
    'description',
    'disabled',
    'layout',
    'max',
    'min',
    'orientation',
    'readout',
    'readout_format',
    'step',
    'style',
    'tabbable',
    'tooltip',
    'value']
```

( While creating a widget, you can set some or all of the initial values of that widget by defining them as keyword arguments in the widgetâĂŹs constructor
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- If you need to display the same value two different ways, youâĂŹll have to use two different widgets. Instead of attempting to manually synchronize the values of the two widgets, you can use the link or jslink function to link two properties together.
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```
[12]: a = widgets.FloatText()
b = widgets.FloatSlider()
display(a,b)
mylink = widgets.jslink((a, 'value'), (b, 'value'))
\(26.9 \quad\) ค
26.90
```

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    26.9 目
    O
```

(1) Unlinking the widgets is simple. All you have to do is call .unlink on the link object.
[13]: \# mylink.unlink()

## Numeric widgets

There are many widgets distributed with ipywidgets that are designed to display numeric values. Widgets exist for displaying integers and floats, both bounded and unbounded. The integer widgets share a similar naming scheme to their floating point counterparts. By replacing Float with Int in the widget name, you can find the Integer equivalent.
(3) The slider is displayed with a specified, initial value. Lower and upper bounds are defined by min and max, and the value can be incremented according to the step parameter.
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```
[2]: widgets.IntSlider(
    value=7,
    min=0,
    max=10,
    step=1,
    description='Test:',
    disabled=False,
    continuous_update=False,
    orientation='horizontal',
    readout=True,
    readout_format='d'
)
```

(10) The FloatLogSlider has a log scale, which makes it easy to have a slider that covers a wide range of positive magnitudes. The min and max refer to the minimum and maximum exponents of the base, and the value refers to the actual value of the slider.
(6) The FloatLogSlider has a log scale, which makes it easy to have a slider that covers a wide range of positive magnitudes. The min and max refer to the minimum and maximum exponents of the base, and the value refers to the actual value of the slider.

```
[5]: widgets.FloatLogSlider(
    value=10,
    base=10,
    min=-10, # max exponent of base
    max=10, # min exponent of base
    step=0.2, # exponent step
    description='Log Slider'
)
```

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    description='Log Slider'
)
```

(1) IntRangeSlider and FloatRangeSlide.

```
[6]: widgets.IntRangeSlider(
    value=[5, 7],
    min=0,
    max=10,
    step=1,
    description='Test:',
    disabled=False,
    continuous_update=False,
    orientation='horizontal',
    readout=True,
    readout_format='d',
)
```

```
[7]: widgets.FloatRangeSlider(
    value=[5, 7.5],
    min=0,
    max=10.0,
    step=0.1,
    description='Test:',
    disabled=False,
    continuous_update=False,
    orientation='horizontal',
    readout=True,
    readout_format='.1f',
)
```


## Outline

## Session XV

(1) Test $1+$ Correction

## Questions+ Solutions

Assume the following list definition:

$$
a=[' f o o ', ~ ' b a r ', ~ ' b a z ', ~ ' q u x ', ~ ' q u u x ', ~ ' c o r g e '] ~
$$

Which display correct output?

Answer
(-) >>> print(a[4::-2])
['quux', 'baz', 'foo']
>>> print(a[-6])
Traceback (most recent call last): File "<stdin>", line 1, in <module> IndexError: list index out of range
(-) >>>print(a[-5:-3])
['bar', 'baz']
$\ggg a[:]$ is a
True

```
List a is defined as follows:
a = ['a', 'b', 'c']
Which of the following statements adds 'd' and 'e' to the end of a, so that it then equals ['a', 'b', 'c', 'd', 'e']:
```


## Answer

(- a.extend(['d', 'e'])
$a[-1:]=[$ 'd', 'e']
a.append(['d', 'e'])
(6) a[len(a):] = ['d', 'e']
$a+=$ 'de'

- $\mathrm{a}+=$ ['d', 'e']

| Question | Suppose s is assigned as follows: |
| :---: | :---: |
|  | $s=\text { 'foobar' }$ <br> All of the following expressions produce the same result except one. Which one? |
| Answer | $s[:-1][\because-5]$ |
|  | (- $s[\because-5]$ |
|  | $s[: 5]$ |
|  | $s[:-1][-1]+s[l e n(s)-1]$ |
|  | $s[0]+s[-1]$ |

## Short Answer: You have a list a defined as follo...

## Question

You have a list a defined as follows:
$a=[1,2,7,8]$
Write a Python statement using slice assignment that will fill in the missing values so that a equals [1, 2, 3, 4, 5, 6, 7, 8].

Answer
$a[2: 2]=[3,4,5,6]$

| Question | Consider this statement: |
| :---: | :---: |
|  | >>> print ('foo <br> bar\nbaz') <br> Which of the following is the correct output? |
| Answer | foo\bar\nbaz |
|  | foolbar baz |
|  | fool\barnbaz |
|  | foollbar\nbaz |

## True/False: Every time when we modify the string...

(6) True

False

True/False: In Python 3, the maximum value for an...

Question

Answer

In Python 3, the maximum value for an integer is $2^{63}-1$ :

True
© False

| Question | What is the output? $\begin{aligned} & \text { for } i \text { in range }(10,15,1) \text { : } \\ & \operatorname{print}(i) \end{aligned}$ |
| :---: | :---: |
| Answer | (-10,11, 12, 13, 14 |
|  | $10,11,12,13,14,15$ |
|  | $9,10,11,12,13,14$ |
|  |  |
| 1. Multiple | utput of this code? $\mathbf{i}=5 .$. |
| Question | What is the output of this code? $\begin{aligned} & i=s=0 \\ & \text { while } \quad i<=3 \text { : } \\ & \quad s+=i \\ & \quad i=i+1 \end{aligned} \text { print(s) }$ |
| Answer | 3 |
|  | 4 |
|  | (6) |
|  | 0 |


| Question | What is the output? ```a=10 if a<5: a=20 elif a>1: a=500 elif a>100: a=1 else: a=0 print(a)``` |
| :---: | :---: |
| Answer | 0 |
|  | 1 |
|  | 10 |
|  | 20 |
|  | (-500 |
|  | None of the above |


| Question | ```What is the output? def f(L): L.append(50) L.append(30) return M=[1,2,3] print(M) f(M) print(M)``` |
| :---: | :---: |
| Answer | $\begin{aligned} & {[1,2,3,50,30]} \\ & {[1,2,3,50,30]} \end{aligned}$ |
|  | $\begin{aligned} & {[1,2,3]} \\ & {[1,2,3]} \end{aligned}$ |
|  | $\begin{aligned} & {[1,2,3]} \\ & {[1,2,3,50,30]} \end{aligned}$ |
|  | $\begin{aligned} & {[1,2,3,50,30]} \\ & {[1,2,3]} \end{aligned}$ |

```
def f(a):
    a+=5
    print(a)
    return a
a=6
print(a)
f(a)
print(a)
```


## Answer

Error
(6) 6116

61111

6 Followed by an error

61166

## 6111111

| Question In Python |  |
| :--- | ---: |
| Answer | True |
|  | False |

## Multiple Answer: List a is defined as follows: $\mathrm{a}=\ldots$

```
Question List a is defined as follows:
a=[1, 2, 3, 4, 5]
Select all of the following statements that remove the middle element 3 from a so that it equals [1, 2, 4, 5]:
Answer a[2] = [ ]
```

(8) $a[2: 3]=[]$
(8) a.remove(3)
$a[2: 2]=[]$

- del a[2]((-%5Cbigcirc=))

| Question | What is the output? $\begin{aligned} & \text { det myst }(a, b): \\ & \\ & a=a+b \\ & b=a-b \\ & a=a-b \\ & \text { return }[a, b] \end{aligned}$ |
| :---: | :---: |
| Answer | [3,5] |
|  | [5,3] |
|  | [6,-2] |
|  | - None of the above |



| Question | What is the result of the following statement: |
| :---: | :---: |
|  | list( ([a,b, c, d, e] + 'fghi') [3:6]) |
| Answer | ['d', 'e', 'f'] |
|  | 'def' |
|  | (\%) It raises an error |
|  | [100, 101, 102] |
|  | ['d', 'e', 'f'] |
| 9. Short Answer: What is the slice expression that giv... |  |
| Question | What is the slice expression that gives every third character of string s, starting with the last character and proceeding backward to the first? |
| Answer | $s[\cdots-3]$ |

A list may contain any type of object except another list

All elements in a list must be of the same typeA given object may appear in a list more than onceThere is no conceptual limit to the size of a list

These represent the same list:
['a', 'b', 'c']
['c', 'a', 'b']

## Multiple Answer: Which of the following are true:

Question
Which of the following are true:

Answer$s[:]==s$$s[\because-1][\because-1]==s$
$s[\because-1][\because-1]$ is $s$
$s[:]$ is $s$


| Question | ```def calculate (num1, num2): res = num1 * num2 return(res) calculate(5, 6)``` |
| :---: | :---: |
| Answer | 20 |
|  | - 30 |
|  | The program executed with errors |
|  |  |
| 5. Multiple Choice: for $x$ in range(0.5, 5.5, 0.5): \&... |  |
| Question | ```for x in range(0.5, 5.5, 0.5): print(x)``` |
| Answer | $[0.5,1,1.5,2,2.5,3,3.5,4,4.5,5,5.5]$ |
|  | $[0.5,1,1.5,2,2.5,3,3.5,4,4.5,5]$ |
|  | ( The Program executed with errors |


| Question | ```sampleList = ["Jon", "Kelly", "Jessa"] sampleList.append(2,"Scott") print(sampleList)``` |
| :---: | :---: |
| Answer | ['Jon', 'Kelly', 'Scott', 'Jessa'] |
|  | ['Jon', 'Kelly', 'Jessa', 'Scott'] |
|  | ['Jon', 'Scott', 'Kelly', 'Jessa'] |
|  | - The program executed with errors |
| . Multiple Choice: var = "James" * 2 * 3 \&nbs... |  |
| Question | $\begin{aligned} & \text { var }=\text { "James" *2 * } 3 \\ & \text { print(var) } \end{aligned}$ |
| Answer | - JamesJamesJamesJamesJamesJames |
|  | JamesJamesJamesJamesJames |
|  | Error: invalid syntax |

$\qquad$

```
Question
var1=1
var2=2
var3 = "3"
print(var + var2 + var3)
```

Answer
6

- Error. Mixing operators between numbers and strings are not supported

33

123

## 9. Multiple Choice: var="James Bond" print(var[2::-1])

| Question | ```var="James Bond" print(var[2::-1])``` |
| :---: | :---: |
| Answer | dnoB semaj |
|  | ( maj |
|  | dno |
|  | Jam |


| Question | $\mathrm{x}=36 / 4^{*}(3+2) * 4+2$ <br> $\operatorname{print}(\mathrm{x})$ |
| :--- | :--- |
| Answer | 182.0 |
|  | 37 |
| 117 |  |
| The Program executed with errors |  |

```
Question What is the output?
    def op1(a, b, c):
        return a + b * c
    def op2(a, b, c):
        return a * 2 + b
    def op3(a, b, c):
        return c * 3 - b
    i = 1
    j = 1
    k = 1
    if i> 5:
        result = op1(i,j,k)
    elif j < 3:
        result =op2(j, k, i)
    else:
        result = op3(k, j, i)
    print (result)
Answer (}
```

    5
    2
    1

## 1. Exercises

## Time format

Given a time in 12-hour AM/PM format, convert it to military (24-hour) time.

## Time format

Given a time in 12-hour AM/PM format, convert it to military (24-hour) time.

## Note

Midnight is 12:00:00 AM on a 12 -hour clock and 00:00:00 on a 24 -hour clock. Noon is 12:00:00 PM on 12-hour clock and 12:00:00 on 24 -hour clock.

## Time format

Given a time in 12-hour AM/PM format, convert it to military (24-hour) time.

## Note

Midnight is 12:00:00 AM on a 12-hour clock and 00:00:00 on a 24 -hour clock. Noon is 12:00:00 PM on 12-hour clock and 12:00:00 on 24 -hour clock.

## Examples:

```
Input : 11:21:30 PM
Output : 23:21:30
Input : 12:12:20 AM
Output : 00:12:20
```


## Solution

```
# Python program to convert time
# from 12 hour to 24 hour format
# Function to convert the date format
def convert24(str1):
# Checking if last two elements of time
# is AM and first two elements are 12
if str1[-2:] == "AM" and str1[:2] == "12":
                return "00" + str1[2:-2]
# remove the AM
elif strl[-2:] == "AM":
        return strl[:-2]
# Checking if last two elements of time
# is PM and first two elements are 12
elif strl[-2:] == "PM" and str1[:2] == "12":
        return str1[:-2]
    else:
```

```
# add 12 to hours and remove PM
```


# add 12 to hours and remove PM

return str(int(str1[:2]) + 12) + str1[2:8]

```
return str(int(str1[:2]) + 12) + str1[2:8]
```


## Second way

Given a time in military (24-hour), convert it to $\mathbf{1 2}$-hour AM/PM formattime.

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Given a time in military (24-hour), convert it to $\mathbf{1 2}$-hour AM/PM formattime.

## Note

Midnight is 00:00:00 on a 24 -hour clock and 12:00:00 AM on a 12-hour clock. Noon is 12:00:00 on 24 -hour clock and 12:00:00 PM on 12-hour clock.

## Second way

Given a time in military (24-hour), convert it to $\mathbf{1 2}$-hour AM/PM formattime.

## Note

Midnight is 00:00:00 on a 24 -hour clock and 12:00:00 AM on a 12-hour clock. Noon is 12:00:00 on 24 -hour clock and 12:00:00 PM on 12-hour clock.

## Examples:

```
Input : 17:35:20
Output : 5:35:20 PM
Input : 00:10:24
Output : 12:10:24 AM
```


## Solution

```
# Convert Function which takes in
# 24hour time and convert it to
# 12 hour format
def convert12(str):
    # Get Hours
    h1 = ord(str[0]) - ord('0');
    h2 = ord(str[1]) - ord('0');
    hh = h1 * 10 + h2;
    # Finding out the Meridien of time
    # ie. AM or PM
    Meridien="";
    if (hh < 12):
        Meridien = "AM";
    else:
        Meridien = "PM";
    hh %= 12;
    # Handle 00 and 12 case separately
    if (hh == 0):
        print("12", end = "");
        # Printing minutes and seconds
        for i in range(2, 8):
            print(str[i], end = "");
    else:
        print(hh,end="");
        # Printing minutes and seconds
        for i in range(2, 8):
            print(str[i], end = "");
    # After time is printed
    # cout Meridien
    print(" " + Meridien);
```


## Is prime?

Given a positive integer, check if the number is prime or not.

## Note

A prime is a natural number greater than 1 that has no positive divisors other than 1 and itself. Examples of first few prime numbers are $\{2,3,5, \ldots$

## Is prime?

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## Note

A prime is a natural number greater than 1 that has no positive divisors other than 1 and itself. Examples of first few prime numbers are $\{2,3,5, \ldots$

## Examples:

Input: $\mathrm{n}=11$
Output: true

Input: $\mathrm{n}=15$
Output: false

Input: $\mathrm{n}=1$
Output: false

## Is prime?

Given a positive integer, check if the number is prime or not.

## Note

A prime is a natural number greater than 1 that has no positive divisors other than 1 and itself. Examples of first few prime numbers are $\{2,3,5, \ldots$

```
def isPrime(n):
    # Corner case
    if n <= 1:
        return False
    # Check from 2 to n-1
    for i in range(2, n):
        if n % i == 0:
            return False;
    return True
# Driver Program to test above function
print("true") if isPrime(11) else print("false")
print("true") if isPrime(14) else print("false")
```


## Find largest prime factor of a number

Given a positive integer $n ;\left(1 \leq n \leq 10^{15}\right)$. Find the largest prime factor of a number. .

Find largest prime factor of a number

Given a positive integer $n ;\left(1 \leq n \leq 10^{15}\right)$. Find the largest prime factor of a number. .

```
Input: 6
Output: 3
Explanation
Prime factor of 6 are- 2, 3
Largest of them is \'3\'
Input: 15
Output: 5
```

Find largest prime factor of a number

Given a positive integer $n ;\left(1 \leq n \leq 10^{15}\right)$. Find the largest prime factor of a number.

```
def maxPrimeFactors(n):
    l=[ ]
    for i in range(2,n):
        if i%n==0:
                                l.append(i)
    p=[ ]
    for i in l:
        if isPrime(i)==True:
        p.append(i)
    return max(p)
```


## 2. Fibbonacci

## Golden number

What do we have by Tool in common with:

## Golden number

What do we have by Tool in common with: sunflowers,


## Golden number

What do we have by Tool in common with: sunflowers, the Golden ratio,


## Golden number

What do we have by Tool in common with: sunflowers, the Golden ratio, fur tree cones,


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What do we have by Tool in common with: sunflowers, the Golden ratio, fur tree cones, The Da Vinci Code


## Golden number

What do we have by Tool in common with: sunflowers, the Golden ratio, fur tree cones, The Da Vinci Code the song "Lateralus".

## "Lateralus"

(1)Black, (1) then,
(1) much,
(2) more that
(2) white are, (3) all I see,
(5) in my infancy,
(8) red and yellow then came to be,
(5) reaching out to me,
(3) lets me see.
(2) There is, (1) so,
(3) beckons me,
(5) to look through to these,
(8) infinite possibilities.
(13) As below so above and beyond I imagine,
(8) drawn outside the lines of reason.
(5) Push the envelope. (3) Watch it bend.

## Golden number

What do we have by Tool in common with: sunflowers, the Golden ratio, fur tree cones, The Da Vinci Code the song "Lateralus".
The Fibonacci numbers are the numbers of the following sequence of integer values: $0,1,1,2,3,5,8,13,21,34,55,89, \ldots$


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$$
F_{n}=F_{n-1}+F_{n-2} \text { with } F_{0}=0 \text { and } F_{1}=1
$$

## Golden number

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$$
F_{n}=F_{n-1}+F_{n-2} \text { with } F_{0}=0 \text { and } F_{1}=1
$$

## History

The Fibonacci sequence is named after the mathematician Leonardo of Pisa, who is better known as Fibonacci. In his book "Liber Abaci" (published 1202) he introduced the sequence as an exercise dealing with bunnies. His sequence of the Fibonacci numbers begins with $F_{1}=1$, while in modern mathematics the sequence starts with $F_{0}=0$. But this has no effect on the other members of the sequence.

## Coding Fibonnaci

Write in two different ways, the functions $f i b(n)$ and $f i b i(n)$. Where the first is written in a reccursive way and the second one with a for Loop.

## Coding Fibonnaci

Write in two different ways, the functions $f i b(n)$ and $f i b i(n)$. Where the first is written in a reccursive way and the second one with a for Loop.

```
def fib(n):
    if n == 0:
    return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```


## Coding Fibonnaci

Write in two different ways, the functions $f i b(n)$ and $f i b i(n)$. Where the first is written in a reccursive way and the second one with a for Loop.

```
def fibi(n):
    a, b = 0, 1
    for i in range(n):
        a, b = b, a + b
    return a
```


## Time comparaison

If you check the functions fib() and fibi() , you will find out that the iterative version fibi()is a lot faster than the recursive version fib(). To get an idea of how much this "a lot faster" can be, we have written a script where we you the timeit module to measure the calls:

## Time comparaison

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```
from timeit import Timer
from fibo import fib
t1 = Timer("fib(10)","from fibo import fib")
for i in range(1,41):
    s = "fib(" + str(i) + ")"
    t1 = Timer(s,"from fibo import fib")
    time1 = t1.timeit(3)
    s = "fibi(" + str(i) + ")"
    t2 = Timer(s,"from fibo import fibi")
    time2 = t2.timeit(3)
    print("n=%2d, fib: %8.6f, fibi: %7.6f, percent: %10.2f" % (i, time1, time2, time1/time2))
```

Change fibo by $\qquad$ main $\qquad$ .

## Time comparaison

## If you check the functions fib() and fibi(), you will find out that the iterative

 version fibi()is a lot faster than the recursive version fib(). To get an idea of how much this "a lot faster" can be, we have written a script where we you the timeit module to measure the calls: Change fibo by $\qquad$ main $\qquad$ .$\left.\begin{array}{llll}\hline n=1, & \text { fib: } & 0.000004, \text { fibi: } & 0.000005, \text { percent: }\end{array}\right) 0.81$

## 1. Widgets

Using widgets
Baby steps.
(1) To use the widget framework, you need to import ipywidgets:

## [1]: import ipywidgets as widgets

(c) Widgets have their own display representation which allows them to be displayed using IPython's display framework. Constructing and returning an IntSlider automatically displays the widget.

```
[2]: widgets.IntSlider()
```

- You can also explicitly display the widget using display(...).
[3]: from IPython.display import display w = widgets. IntSlider() display(w)
(0) The FloatLogSlider has a log scale, which makes it easy to have a slider that covers a wide range of positive magnitudes. The min and max refer to the minimum and maximum exponents of the base, and the value refers to the actual value of the slider.

```
[5]: widgets.FloatLogSlider(
    value=10,
    base=10,
    min=-10, # max exponent of base
    max=10, # min exponent of base
    step=0.2, # exponent step
    description='Log Slider'
)
```

(0) IntRangeSlider and FloatRangeSlide.

```
[6]: widgets.IntRangeSlider(
    value=[5, 7],
    min=0,
    max=10,
    step=1,
    description='Test:',
    disabled=False,
    continuous_update=False,
    orientation='horizontal',
    readout=True,
    readout_format='d',
)
```

```
[7]: widgets.FloatRangeSlider(
    value=[5, 7.5],
    min=0,
    max=10.0,
    step=0.1,
    description='Test:',
    disabled=False,
    continuous_update=False,
    orientation='horizontal',
    readout=True,
    readout_format='.1f',
)
```

- IntProgress and FloatProgress.


## - IntProgress and FloatProgress.

```
[8]: widgets.IntProgress(
    value=7,
    min=0,
    max}=10\mathrm{ ,
    step=1,
    description='Loading: ',
    bar_style='', # 'success', 'info', 'warning', 'danger' or ''
    orientation='horizontal'
)
```

    [9]: widgets.FloatProgress(
        value=7.5,
        min=0,
        \(\max =10.0\),
        step=0.1,
        description='Loading:',
        bar_style='info',
        orientation='horizontal'
    )
(0) BoundedlntText and BoundedFloatText.
(0) BoundedIntText and BoundedFloatText.
[10]: widgets.BoundedIntText( value=7, min=0, $\max =10$, step=1, description='Text:', disabled=False
)
[11]: widgets.BoundedFloatText( value=7.5, min=0, $\max =10.0$, step $=0.1$, description='Text:', disabled=False )

## Boolean widgets

There are three widgets that are designed to display a boolean value.

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© ToggleButton.

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There are three widgets that are designed to display a boolean value.
( 3 ToggleButton .
[14]: widgets.ToggleButton(
value=False,
description='Click me',
disabled=False,
button_style='', \# 'success', 'info', 'warning', 'danger' or '' tooltip='Description',
icon='check' \# (FontAwesome names without the 'fa-" prefix)
)

## Boolean widgets

There are three widgets that are designed to display a boolean value.
( 3 ToggleButton .

```
[14]: widgets.ToggleButton(
    value=False,
    description='Click me',
    disabled=False,
    button_style='', # 'success', 'info', 'warning', 'danger' or ''
    tooltip='Description',
    icon='check' # (FontAwesome names without the 'fa-' prefix)
)
```

- Valid.

The valid widget provides a read-only indicator.

## Boolean widgets

There are three widgets that are designed to display a boolean value.
( 3 ToggleButton .

```
[14]: widgets.ToggleButton(
    value=False,
    description='Click me',
    disabled=False,
    button_style='', # 'success', 'info', 'Warning', 'danger' or ''
    tooltip='Description',
    icon='check' # (FontAwesome names without the 'fa-` prefix)
)
```

- Valid.

The valid widget provides a read-only indicator.

```
[16]: widgets.Valid(
        value=False,
        description='Valid!',
)
```

(1) Checkbox.
(1) Checkbox.

- value specifies the value of the checkbox
- indent parameter places an indented checkbox, aligned with other controls. Options are True (default) or False.

```
[15]: widgets.Checkbox(
    value=False,
    description='Check me',
    disabled=False,
    indent=False
)
```


## Selection widget

Widgets used to display a selection list.
(1) Dropdown.

## Selection widget

Widgets used to display a selection list.
(1) Dropdown.

```
[17]: widgets.Dropdown(
    options=['1', '2', '3'],
    value='2',
    description='Number:',
    disabled=False,
)
```


## Selection widget

Widgets used to display a selection list.
(1) Dropdown.
[17]: widgets.Dropdown(
options=['1', '2', '3'], value='2',
description='Number:',
disabled=False,
)
[18]: widgets.Dropdown(
options=[('One', 1), ('Two', 2), ('Three', 3)], value=2,
description='Number: ',
)
(13) RadioButtons.

## (13) RadioButtons.

[19]: widgets.RadioButtons(
options=['pepperoni', 'pineapple', 'anchovies'],
\# value='pineapple', \# Defaults to 'pineapple'
\# layout=\{'width': 'max-content'\}, \# If the items' names are long
description='Pizza topping:',
disabled=False
)

## (13) RadioButtons.

```
[19]: widgets.RadioButtons(
    options=['pepperoni', 'pineapple', 'anchovies'],
        # value='pineapple', # Defaults to 'pineapple'
# layout={'width': 'max-content'}, # If the items' names are long
    description='Pizza topping:',
    disabled=False
)
```

(3) SelectMultiple.
(12) RadioButtons.

```
[19]: widgets.RadioButtons(
    options=['pepperoni', 'pineapple', 'anchovies'],
# value='pineapple', # Defaults to 'pineapple'
# layout={'width': 'max-content'}, # If the items' names are long
    description='Pizza topping:',
    disabled=False
)
```

(3) SelectMultiple.

Multiple values can be selected with shift and/or ctrl (or command) pressed and mouse clicks or arrow keys.

```
[25]: widgets.SelectMultiple(
    options=['Apples', 'Oranges', 'Pears'],
    value=['Oranges'],
    #rows=10,
    description='Fruits',
    disabled=False
)
```

(5) Horizontal Buttons.
(5) Horizontal Buttons.

```
color_buttons = widgets.ToggleButtons(
    options=['blue', 'red', 'green'],
    description='Color:',
)
color_buttons
```

Color: blue red green

## Password format

The Password widget hides user input on the screen.

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The Password widget hides user input on the screen. This widget is not a secure way to collect sensitive information because:

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- The contents of the Password widget are transmitted unencrypted.


## Password format

The Password widget hides user input on the screen.
This widget is not a secure way to collect sensitive information because:

- The contents of the Password widget are transmitted unencrypted.
- If the widget state is saved in the notebook the contents of the Password widget is stored as plain text.
[29]: widgets.Password(
value='password',
placeholder='Enter password', description='Password:' ,
disabled=False
)


## Date and Color picker

The date picker widget works in Chrome, Firefox and IE Edge, but does not currently work in Safari.

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The date picker widget works in Chrome, Firefox and IE Edge, but does not currently work in Safari.

```
[36]: widgets.DatePicker(
    description='Pick a Date',
    disabled=False
)
```


## Date and Color picker

The date picker widget works in Chrome, Firefox and IE Edge, but does not currently work in Safari.

```
[36]: widgets.DatePicker(
    description='Pick a Date',
    disabled=False
)
```

[37]: widgets.ColorPicker(
concise=False,
description='Pick a color',
value='blue',
disabled=False
)

## Play animation widget

The Play widget is useful to perform animations by iterating on a sequence of integers with a certain speed. The value of the slider below is linked to the player.

## Play animation widget

The Play widget is useful to perform animations by iterating on a sequence of integers with a certain speed. The value of the slider below is linked to the player.

```
[35]: play = widgets.Play(
    value=50,
    min=0,
    max=100,
    step=1,
    interval=500,
    description="Press play",
    disabled=False
)
slider = widgets.IntSlider()
widgets.jslink((play, 'value'), (slider, 'value'))
widgets.HBox([play, slider])
```


## 2. Interactive section

## Interact

The interact function (ipywidgets.interact) automatically creates user interface controls for exploring code and data interactively.

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```
[1]: from __future__ import print_function
from ipywidgets import interact, interactive, fixed, interact_manual
import ipywidgets as widgets
```


## Interact

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```
[1]: from __future__ import print_function
from ipywidgets import interact, interactive, fixed, interact_manual
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To use interact, you need to define a function that you want to explore.

## Interact

The interact function (ipywidgets.interact) automatically creates user interface controls for exploring code and data interactively.

```
[1]: from __future__ import print_function
from ipywidgets import interact, interactive, fixed, interact_manual
import ipywidgets as widgets
```

To use interact, you need to define a function that you want to explore.

```
In [17]: def f(x):
    return 4*x-3
interact(f,x=2);
```

x
2
5

## Several arguments

In [17]: $\operatorname{def} f(x)$ :
return $4^{*} x-3$
interact ( $\mathrm{f}, \mathrm{x}=2$ ) ;
x


2
5

In [18]: interact( $f, x=$ widgets. IntSlider(min=-10, max=30, step=1, value=10));


93

In [19]: interact(f, $x=(0,8,2))$;
x


4
13

In [21]: interact(f, $x=[(' o n e ', ~ 10), ~(' t w o ', ~ 20)]) ;$


37

## Interactive function

In addition to interact, IPython provides another function, interactive, that is useful when you want to reuse the widgets that are produced or access the data that is bound to the controls.

## Interactive function

In addition to interact, IPython provides another function, interactive, that is useful when you want to reuse the widgets that are produced or access the data that is bound to the controls.

```
18]: from IPython.display import display
def f(a, b):
        display(a + b)
        return a+b
```

[19]: $w=$ interactive(f, $\mathrm{a}=10, \mathrm{~b}=20$ )
30
[22]: display(w)
a10
b ..... 20

## Interactive_output

interactive output provides additional flexibility: you can control how the elements are laid out.

## Interactive_output

interactive output provides additional flexibility: you can control how the elements are laid out.

```
[31]: a = widgets.IntSlider()
b = widgets.IntSlider()
c = widgets.IntSlider()
ui = widgets.HBox([a, b, c])
def f(a, b, c):
    print((a, b, c))
out = widgets.interactive_output(f, {'a': a, 'b': b, 'c': c})
display(ui, out)
(0, 0, 0)
```


## Example

```
[33]: %matplotlib inline
from ipywidgets import interactive
import matplotlib.pyplot as plt
import numpy as np
def f(m,b):
    plt.figure(2)
    x = np.linspace(-10, 10, num=1000)
    plt.plot(x, m * x + b)
    plt.ylim(-5, 5)
    plt.show()
interactive_plot = interactive(f, m=(-2.0, 2.0), b=(-3,3, 0.5))
output = interactive_plot.children[-1]
output.layout.height = '350px'
interactive_plot
```


## Example



## Interactive functions

Here is an example joining the widgets with matplotlib;

## Interactive functions

Here is an example joining the widgets with matplotlib;

```
In [26]: import numpy as np
import matplotlib.pyplot as plt
@widgets.interact manual(
    color=['blue', 'red', 'green'], lw=(1., 10.))
def plot(freq=1., color='blue', lw=2, grid=True):
    t = np.linspace(-1., +1., 1000)
    fig, ax = plt.subplots(1, 1, figsize=(8, 6))
    ax.plot(t, np.sin(2 * np.pi * freq * t),
            lw=lw, color=color)
    ax.grid(grid)
```


## Interactive functions

Here is an example joining the widgets with matplotlib;


Run Interact


## 3. More exercises

## Perfect numbers

In number theory, a perfect number is a positive integer that is equal to the sum of its proper positive divisors, that is, the sum of its positive divisors excluding the number itself.

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The first perfect number is 6 , because 1,2 , and 3 are its proper positive divisors, and $1+2+3=6$. The next perfect number is $28=1+2+4+7+14$. This is followed by the perfect numbers 496 and 8128 .

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```
1 def perfect_number(n):
    sum \(=0\)
    for \(x\) in range(1, n):
        if \(n \% \times=0\) :
        sum += x
    return sum == n
    print(perfect_number(6))
```


## Palindrome

A palindrome is nothing but any number or a string which remains unaltered when reversed.

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## Example: 12321

Output: Yes, a Palindrome number
Example: RACECAR
Output: Yes, a Palindrome string

## Palindrome

A palindrome is nothing but any number or a string which remains unaltered when reversed.

```
string=input(("Enter a string:"))
    if(string==string[::-1]):
    print("The string is a palindrome")
    else:
    print("Not a palindrome")
```

Given an array of positive integers. All numbers occur even number of times except one number which occurs odd number of times.

Given an array of positive integers. All numbers occur even number of times except one number which occurs odd number of times.

## Examples:

```
Input : arr = {1, 2, 3, 2, 3, 1, 3}
Output : 3
Input : arr = {5, 7, 2, 7, 5, 2, 5}
Output : 5
```

Given an array of positive integers. All numbers occur even number of times except one number which occurs odd number of times.
> \# function to find the element occurring odd \# number of times def getOddOccurrence(arr, arr_size):

```
for i in range(0, arr_size):
    count = 0
    for j in range(0, arr_size):
        if arr[i] == arr[j]:
        count+= 1
```

    if (count \% 2 != 0):
        return arr[i]
    return -1

